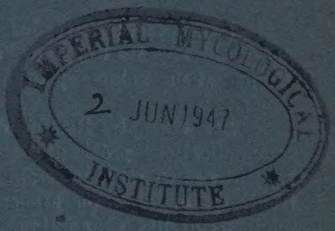
COMPTES RENDUS
(DOKLADY)
DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1941



VOLUME XXXI

№ 3

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
MOSCOU

AVIS AUX AUTEURS

Les «Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS» paraissent tous les dix jours. Chaque numéro a 96 pages en moyenne. Neuf numéros composent un volume pourvu d'une table des matières. Il y a quatre volumes par année.

Les «Comptes Rendus de l'Académie des Sciences de l'URSS» en de courtes communications précisent l'essentiel des travaux en préparation. Ces articles, ne dépassant pas en règle générale quatre pages, sont donnés en l'une des trois langues: française, anglaise ou allemande. Ils embrassent les sciences mathématiques, physiques, naturelles et appliquées, mettant au jour les résultats de recherches scientifiques en cours ou qui viennent d'être achevées, et représentant ainsi les plus récentes données de l'investigation scientifique.

L'auteur a droit à 50 exemplaires de son article.

Adressez les manuscrits à la Rédaction des «Comptes Rendus», Moscou 19, Volkhonka 18, chambre 15.

30 Avril

COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

COMITÉ DE RÉDACTION

A. Borissiak, de l'Académie, A. Deborin, de l'Académie, A. Fersman, de l'Académie, A. Frumkin, de l'Académie, A. Kolmogoroff, de l'Académie, A. Richter, de l'Académie, S. Wawilow, de l'Académie

NOUVELLE SÉRIE

9-me année

Paraissant tous les dix jours

1941

VOLUME XXXI, № 3

TABLE DES MATIÈRES

Pages

MATHÉMATIQUES

G. Pfeiffer, de l'Académie des Sciences de l'Ukraine **(Г. Пфейффер)**. Sur les équations, les systèmes d'équations aux dérivées partielles du premier ordre à plusieurs fonctions inconnues, qui possèdent une intégrale de S. Lie généralisée 195

A. V. Tovbin (А. Товбин). Sur l'existence du centre des groupes infinis et finis 198

I. L. Lanzewizky (И. Л. Ланцевицкий). Über die Orthogonalität der Féjèr-Szegöschen Polynome 199

G. Mirakyan (Г. Миракьян). Approximation des fonctions continues au moyen de polynômes de la forme $e^{-nx}\sum_{k=0}^{m} C_{k,n}x^k$ 201

I. Ogniewetzki (И. Огиевецкий). Une généralisation d'un théorème de M. J. Hadamard sur la convergence des séries 206

G. A. Brodsky (Г. А. Бродский). Über eine Grenzwertaufgabe der Theorie der biharmonischen Funktionen 208

S. M. Nikolsky (С. М. Никольский). Estimations of the Remainder of Féjèr's Sum for Periodical Functions Possessing a Bounded Derivative 210

S. M. Nikolsky (С. М. Никольский). An Asymptotic Estimation of the Remainder under Approximation by Interpolating Trigonometric Polynomials 215

PHYSIQUE

E. Gross and **A. Siromyatnikov (Е. Гросс и А. Сыромятников)**. Structure of the Rayleigh Line and Viscosity of Liquids 219

W. W. Wladimirsky (В. В. Владимирский). Über die Drehung der Polarisationsebene im gekrümmten Lichtstrahl 222

P. Meikljar (П. Мейкляр). Ein Analogon des photographischen Schwarzschild-Effektes in reinem Chlorsilberkristall 226

N. Tolstoy and **V. Tsvetkov (Н. Толстой и В. Цветков).** The Electro-Optical Effect of Kerr in Solutions of *p*-Azoxyanisol 230
A. M. Didenko et N. L. Pokrovsky (А. М. Диденко и Н. Л. Покровский). Sur la mesure du coefficient de température de la tension superficielle du mercure 233

PHYSIQUE TECHNIQUE

V. S. Nesterov (В. С. Нестеров). Sound Absorption by Multilayer Resonance Systems 237

GÉOLOGIE

V. V. Beloussov (В. В. Белоусов). Gravitational Anomalies and Processes of Tectogenesis 241

MINÉRALOGIE

N. A. Eliseev and **E. A. Sverjinskaya (Н. А. Елисеев и Е. А. Свержинская).** Chemical Composition of Lovozero Eudialytites 244
G. P. Barsanov (Г. П. Барсанов). On the Mineralogy of the Iona Iron-Ore Deposit 246

PÉTROGRAPHIE

V. P. Florensky (В. П. Флоренский). Fluorite in the Lower Permian Deposits of the Bashkir Autonomous Socialist Soviet Republic 251

GÉOPHYSIQUE APPLIQUÉE

A. A. Petrowsky and **B. N. Dostovalov (А. А. Петровский и Б. Н. Достовалов).** The Method of Horizontal Antenna as Applied to the Measurement of the Glacial Cover 255

BIOCHIMIE

I. J. Pritzker (И. Я. Прицкер). Über die Ablagerung von Fett in der Schale von Vogeleiern 258

GÉNÉTIQUE

E. N. Volotov (Е. Н. Волотов). Polyploids in *Papaver somniferum* L. Induced by Treatment with Colchicine 261
B. N. Sidorov and **N. N. Sokolov (Б. Н. Сидоров и Н. Н. Соколов).** Production of Tetraploids in *Ricinus communis* Treated with Colchicine 264
J. A. Rapoport (И. А. Рапопорт). Mutations Restituting the Telomere . 266
J. A. Rapoport (И. А. Рапопорт). On Roentgenomutations in Repetitions and X-Ray Mutations as Conditioned by Breaks 270

PHYSIOLOGIE VÉGÉTALE

A. A. Kuzmenko, L. L. Seredenko and **V. A. Goiko (А. А. Кузьменко, Л. Л. Середенко и В. А. Гойко).** Ontogenetic Development and Yield of Cotton as Influenced by the Proportion of Mineral Elements in the Nutrition 273
A. I. Cheliadinova (А. И. Челядинова). Influence of Illumination Intensity upon the Response of Peanuts to Daylength 276
S. O. Grebinsky (С. О. Гребинский). Carbohydrate Metabolism in Alpine Grasses in Connexion with the Adaptation Problem 279

ICHTHYOLOGIE

K. Golovinskaya (К. Головинская). Some Regularities of Growth and Formation of External Characters in the Cultivated Carp 282

MICROBIOLOGIE

W. E. Altergot, K. S. Lavyghina and **O. P. Kuvshinova (В. Е. Альтергот, К. С. Лавыгина и О. П. Кувшинова).** Destructive Changes of the Protoplasm in the Course of Lysis in the Species *Fusarium* . . . 286
N. N. Sushkina (Н. Н. Сушкина). Development of Azotobacter in the Soils of Arid Steppes and Deserts 290

PALÉONTOLOGIE

J. N. Popov (Ю. Н. Попов). Heteropolar Division of the Suture Line in Some of the Lower Triassic Ammonites 293

Edition de l'Académie des Sciences de l'URSS

Moscou 1941

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

MATHÉMATIQUES

SUR LES ÉQUATIONS, LES SYSTÈMES D'ÉQUATIONS AUX DÉRIVÉES PARTIELLES DU PREMIER ORDRE À PLUSIEURS FONCTIONS INCONNUES, QUI POSSÈDENT UNE INTÉGRALE DE S. LIE GÉNÉRALISÉE

Par G. PFEIFFER, de l'Académie des Sciences de l'Ukraine

Sous l'intégrale de S. Lie généralisée du rang ρ, de la classe $n-1$ [1], nous comprenons un système des connexions

$$\Phi_i(\varphi_1, \dots, \varphi_\rho, \varphi_{\rho+1}, \dots, \varphi_{\rho+n-1}) = 0 \quad i = 1, 2, \dots, \rho \tag{1}$$

ce qui est la même chose

$$\varphi_i = F_i(\varphi_{\rho+1}, \dots, \varphi_{\rho+n-1}) \quad i = 1, 2, \dots, \rho \tag{2}$$

Φ, F — fonctions arbitraires des arguments

$$\varphi \equiv \varphi(z_1, \dots, z_k, x_1, \dots, x_n) \tag{3}$$

$z_1, \dots, z_k$ — fonctions inconnues, $x_1, \dots, x_n$ — variables indépendantes

$$\rho \leqslant k. \tag{4}$$

Aux relations (2) correspondent les équations linéaires en jacobiens, qui satisfont aux conditions d'Hamburger

$$Mi + \sum_{\substack{\sigma=1 \\ \tau}}^{kn} M\overset{\sigma}{i}_{\tau} \frac{\partial z_\sigma}{\partial x_\tau} + (H) = 0 \qquad i = 1, 2, \dots \rho \ [2]. \tag{5}$$

Les systèmes jacobiens d'équations linéaires [3]

$$\left.\begin{aligned} &\sum_{\tau=1}^{n} a_\tau p_{1\tau} - b_1 = 0 \\ &\dots\dots\dots\dots \\ &\sum_{\tau=1}^{n} a_\tau p_{k\tau} - b_k = 0 \\ &p_{\sigma\tau} = \frac{\partial z_\sigma}{\partial x_\tau} \end{aligned}\right\} \tag{6}$$

disposés sur l'équation

$$M1 + \sum_{\substack{\sigma=1 \\ \tau}}^{kn} M\overset{\sigma}{1}_{\tau} \frac{\partial z_\sigma}{\partial x_\tau} + (H) = 0 \tag{7}$$

sont en nombre—k:

$$\left.\begin{aligned} M1b_1 + \sum_{\tau=1}^{n}\Big(\sum_{\sigma=1}^{k} \overset{\sigma}{\underset{\tau}{M1}} b_\sigma\Big)\, p_{1\tau} = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ M1b_k + \sum_{\tau=1}^{n}\Big(\sum_{\sigma=1}^{k} \overset{\sigma}{\underset{\tau}{M1}} b_\sigma\Big)\, p_{k\tau} = 0 \end{aligned}\right\} \tag{8}$$

les coefficients

$$b_1, b_2, \ldots, b_k \tag{9}$$

restent arbitraires.

Les systèmes jacobiens (6), disposés sur les équations

$$\left.\begin{aligned} M1 + \sum_{\sigma\tau}^{kn} \overset{\sigma}{\underset{\tau}{M1}} \frac{\partial z_\sigma}{\partial x_\tau} + (H) = 0 \\ M2 + \sum_{\sigma\tau}^{kn} \overset{\sigma}{\underset{\tau}{M2}} \frac{\partial z_\sigma}{\partial x_\tau} + (H) = 0 \end{aligned}\right\} \tag{10}$$

sont en nombre $-(k-1)$; disposés sur les équations

$$\left.\begin{aligned} M1 + \sum_{\sigma\tau}^{kn} \overset{\sigma}{\underset{\tau}{M1}} \frac{\partial z_\sigma}{\partial z_\tau} + (H) = 0 \\ M2 + \sum_{\sigma\tau}^{kn} \overset{\sigma}{\underset{\tau}{M2}} \frac{\partial z_\sigma}{\partial x_\tau} + (H) = 0 \\ M3 + \sum_{\sigma\tau}^{kn} \overset{\sigma}{\underset{\tau}{M3}} \frac{\partial z_\sigma}{\partial x_\tau} + (H) = 0 \end{aligned}\right\} \tag{11}$$

en nombre $-(k-2)$; et ainsi de suite.

En définitive nous avons:

1. *Sur le système des* ρ *équations* (5), *à l'intégrale générale* (1)—(2), *se trouvent*

$$\sigma = k - \rho + 1 \tag{12}$$

systèmes jacobiens (6); *leur intégration, de chacun à part, est équivalente à l'intégration d'une équation linéaire, homogène à une fonction inconnue; désignons-les par*

$$\Sigma_1, \Sigma_2, \ldots, \Sigma_{k-\rho+1}. \tag{13}$$

En réunissant les équations (13), *nous recevrons le système d'équations linéaires, homogènes*

$$\Sigma \tag{14}$$

avec les intégrales

$$\varphi_1, \ldots, \varphi_\rho, \varphi_{\rho+1}, \ldots, \varphi_{\rho+n-1}. \tag{15}$$

Sous l'intégrale de S. Lie généralisée du rang ρ, de la classe $g < n-1$ (4), nous comprenons un système des connexions

$$\Phi_i(\varphi_1, \ldots, \varphi_\rho, \varphi_{\rho+1}, \ldots, \varphi_{\rho+g}) - 0, \quad i = 1, 2, \ldots, \rho \tag{16}$$

ce qui est la même chose

$$\varphi_i = F_i(\varphi_{\rho+1}, \ldots, \varphi_{\rho+g}), \quad i = 1, 2, \ldots, \rho. \tag{17}$$

Aux relations (17) correspondent les systèmes d'équations linéaires en jacobiens, qui satisfont aux conditions d'Hamburger généralisées

$$\left.\begin{aligned} Mi+\sum_{\sigma=z}^{kg} \overset{\sigma}{Mi}_{z}\frac{\partial z_\sigma}{\partial x_z}+\sum_{\sigma}^{k}\overset{\sigma}{Mi}_{g+1}\frac{\partial z_\sigma}{\partial x_{g+1}}+(H)=0 \\ Nih+\sum_{\sigma=z}^{kg}\overset{\sigma}{Nih}_{z}\frac{\partial z_\sigma}{\partial x_z}+\sum_{\sigma}^{k}\overset{\sigma}{Nih}_{g+1}\frac{\partial z_\sigma}{\partial x_h}+(H)=0 \\ h=g+2,\ g+3,\ \dots,n \end{aligned}\right\} \tag{18}$$

A_i)

$$\left.\begin{aligned} Nih=Mi\,\omega_{ih};\ \overset{\sigma}{Nih}_{z}=\overset{\sigma}{Mi}_{z}\omega_{ih}-\overset{\sigma}{Mi}_{g+1}\omega_{ihz}; \\ \overset{\sigma}{Nih}_{g+1}=\overset{\sigma}{Mi}_{g+1};\quad i=1,2,\dots,\rho\ (^5) \end{aligned}\right\} \tag{19}$$

Les systèmes jacobiens généralisés d'équations linéaires (6)

$$\left.\begin{aligned} ap_{1j}+\sum_{z=1}^{q}a_z^j p_{1z}-b_1^j=0 \\ \dots\dots\dots\dots \\ ap_{kj}+\sum_{z=1}^{q}a_z^j p_{kz}-b_k^j=0 \\ j=g+1,\quad g+2,\dots,n \end{aligned}\right\} \tag{20}$$

disposés sur le système A_1) sont en nombre $-k$; sur les systèmes A_1), A_2) — en nombre $-(k-1)$; sur les systèmes A_1), A_2), A_3) — en nombre $-(k-2)$; et ainsi de suite.

En définitive nous avons

2. *Sur le système des* $\rho(n-g)$ *équations* (18), (19), *à l'intégrale générale* (16) — (17), *se trouvent*

$$\sigma=k-\rho+1 \tag{21}$$

systèmes jacobiens généralisés (20); *leur intégration, de chacun à part, est équivalente à l'intégration d'un système d'équations linéaires, homogènes à une fonction inconnue; désignons-les par*

$$\Sigma_1, \Sigma_2, \dots, \Sigma_{k-\rho+1}. \tag{22}$$

En réunissant les systèmes (22), *nous recevrons le système d'équations linéaires, homogènes*

$$\Sigma \tag{23}$$

avec les intégrales

$$\varphi_1, \dots, \varphi_\rho, \varphi_{\rho+1}, \dots, \varphi_{\rho+g}. \tag{24}$$

Manuscrit reçu
le 15. I. 1941.

LITTÉRATURE CITÉE

[1] Ю. П ф е й ф е р, Ж. ін-та мат. АН УРСР, № 3 (1936). [2] M. H a m b u r g e r, Crelle J., **100** (1887); G. P f e i f f e r, C. R. Acad. Sci. de Paris, **185** (1927); Ю. П ф е й ф е р, Ж. ін-та мат. ВУАН, № 1 (1934). [3] C. J a c o b i, Ges. W., IV; Crelle J., **2**, **23**; Ю. П ф е й ф е р, Ж. мат. цикл. ВУАН, I, вып. 4 (1934). [4] Ю. П ф е й ф е р, Ж. ин-та мат. АН УРСР, № 3 (1936). [5] G. P f e i f f e r, C. R. Acad. Sci. de Paris, **185** (1927); Ю. П ф е й ф е р, Ж. ін-та Мат. ВУАН, № 1 (1934). [6] N. S a l t y k o w, J. de Math. p. et appl., III, sér. 5 (1897); Ю. П ф е й ф е р, Ж. мат. цикл. ВУАН, I, вып. 4 (1934).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

MATHÉMATIQUES

SUR L'EXISTENCE DU CENTRE DES GROUPES INFINIS ET FINIS

Par A. V. TOVBIN

(Présenté par O. J. Schmidt, de l'Académie, le 3. II. 1941)

Théorème 1. *Si un groupe* $\mathfrak{B}$ *possède un diviseur normal* $\mathfrak{B}_1$, *d'indice fini et de centre non trivial possédant au moins un élément d'ordre p* (*p étant un nombre premier*), *alors le groupe* $\mathfrak{B}$ *possède un diviseur normal d'Abel d'ordre fini égal à* p^{α}.

En effet, si A est un élément du centre de $\mathfrak{B}_1$ d'ordre p, tous les éléments $A_1 \dots A_k$ conjugués à A par rapport à $\mathfrak{B}$ appartiennent aussi au centre de $\mathfrak{B}_1$ (parce que le centre d'un diviseur normal est encore un diviseur normal).

Donc, le produit $\{A\}\{A_1\} \dots \{A_k\} = \mathfrak{G}$ sera un groupe d'ordre p^{α} qui est un diviseur normal d'Abel pour $\mathfrak{B}$. Le théorème 1 a pour conséquence le théorème 2.

Théorème 2. *Si un diviseur normal* $\mathfrak{B}_1$ *d'indice fini* p^{α} *d'un groupe* $\mathfrak{B}$ *possède dans son centre un élément d'ordre p, alors le groupe* $\mathfrak{B}$ *possède dans son centre au moins un élément d'ordre p.*

En vertu du théorème 1 le groupe $\mathfrak{B}$ possède un diviseur normal d'ordre p^{β}. Désignons ce diviseur par $\mathfrak{G}$. En décomposant le groupe $\mathfrak{G}$ en classes d'éléments conjugués par rapport au groupe $\mathfrak{B}$, nous remarquons qu'au moins l'une des classes, différente de l'unité, contient un seul élément.

Comme application du théorème 2 aux groupes nommés *p*-groupes, nous obtenons le théorème Dietzman [1]. *Si un p-groupe est de classe finie, il possède un centre non trivial.*

Ou, ce qui est le même:

Si un p-groupe $\mathfrak{B}$ *possède un sous-groupe* $\mathfrak{B}_1$ *d'indice fini et de centre non trivial, il possède lui-même un centre non trivial.*

En effet, le groupe $\mathfrak{B}_1$ contient un diviseur normal R (du groupe $\mathfrak{B}$) d'indice fini dans $\mathfrak{B}$; donc, en vertu des propriétés connues du groupe fini $\frac{\mathfrak{B}}{R}$ on peut former une chaîne de groupes $\mathfrak{B}, \mathfrak{B}', \mathfrak{B}'', \dots$ telle que chaque groupe suivant sera un diviseur normal du groupe précédent. Il en résulte en vertu du théorème 2 que $\mathfrak{B}^{(n)}, \mathfrak{B}^{(n-1)}, \dots, \mathfrak{B}'$, donc $\mathfrak{B}$ aussi, possèdent des centres non triviaux, c. q. f. d.

Manuscrit reçu
le 4. II. 1941.

LITTÉRATURE CITÉE

[1] Dietzmann, C. R. Acad. Sci. URSS, XV, № 2 (1937). [2] Труды семинара по теории групп, 30. [3] Сб. пам. акад. Д. А. Граве, 63.

Traduit par N. Bary.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

MATHEMATIK

ÜBER DIE ORTHOGONALITÄT DER FÉJÈR-SZEGÖSCHEN POLYNOME

Von I. L. LANZEWIZKY

(Vorgelegt von S. N. Bernstein, Mitglied der Akademie, den 14. I. 1941)

Wir verdanken Herrn L. Féjèr [1] die folgende Verallgemeinerung der Legendreschen Polynome. Es sei $f(z)=\sum_{k=0}^{\infty} a_k z^k$ eine im Einheitskreise reguläre Funktion mit lauter reellen und nichtverschwindenden Koeffizienten. Nun bilden wir die Reihenentwicklung

$$|f(re^{i\vartheta})|^2=\sum_{n=0}^{\infty} P_n(\cos\vartheta)\, r^n \qquad (r<1) \tag{1}$$

wo $P_n(\cos\vartheta)=\sum_{k=0}^{\left[\frac{n}{2}\right]}{}' a_k a_{n-k}\cos(n-2k)\vartheta$ ist. Das Zeichen $\sum'$ bedeutet, dass das letzte Glied den Faktor $\frac{1}{2}$ enthält, wenn n eine gerade Zahl ist.

Insbesondere gehen $P_n(\cos\vartheta)$ bei $f(z)=\frac{1}{\sqrt{1-z}}$ in die Legendreschen Polynome über. Im allgemeinen Falle wurden die in der Reihenentwicklung (1) vorkommenden Polynome von Féjèr [1] und Szegö [2] behandelt, die für sie Abschätzungen, die den Abschätzungen von T. I. Stieltjes für die Legendreschen Polynome ähnlich sind, asymptotische Formel und die Nullstellenverteilung gewonnen haben.

Nun stellen wir in dieser Arbeit die Aufgabe, *die notwendigen und hinreichenden Bedingungen dafür zu finden, dass die Polynome* $P_n(x)$, *die zu einer Funktion* $f(z)=\sum_{k=0}^{\infty} a_k z^k$ *gehören, ein Orthogonalsystem bilden*, *d. h. dass die Beziehung* $\int_{-\infty}^{+\infty} P_n(x)\,P_m(x)\,d\psi(x)=\begin{cases}0 & m\neq n\\ h_n>0 & m=n\end{cases}$ *gültig sei.*

Hier ist $\psi(x)$ eine beschränkte, nicht abnehmende Funktion mit unendlich vielen Wachstumstellen. Wollen wir den folgenden Satz beweisen.

Satz 1. *Damit ein Féjèr-Szegösche Polynomsystem, das einer Funktion* $f(z)=\sum_{k=0}^{\infty} a_k z^k$ *gehört, orthogonal sei, ist es notwendig und hinrei-*

chend, dass die Entwicklungskoeffizienten α_k der folgenden Differenzengleichung genügen:

$$C_{n+1} = \frac{C_1 C_n - C_2 C_{n-1}}{C_1 + C_n - C_2 - C_{n-1}} \qquad (n = 3, 4, 5 \ldots) \tag{2}$$

wo $C_n = \frac{\alpha_n}{\alpha_{n-1}}$ *ist.*

Der Beweis ist darauf gegründet, dass für die Orthogonalität eines Polynomsystems der Gestalt $Q_n(x) = x^n + d_1^{(n)} x^{n-1} + \ldots \qquad (n = 0, 1, 2 \ldots)$ notwendig und hinreichend ist, dass die Rekursionsbeziehung

$$Q_{n+1}(x) = (x - \alpha_n) Q_n(x) - \lambda_n Q_{n-1}(x) \qquad (\lambda_n > 0) \tag{3}$$
$$(n = 0, 1, 2, \ldots ; \; Q_{-1}(x) \equiv 0)$$

gültig sei.

Die Lösung der Differenzengleichung (2) ist

$$C_{n+1} = C_2 + (C_2 - C_1) \frac{U_{n-2}(\xi)}{U_n(\xi)} \qquad (n = 3, 4, \ldots) \tag{4}$$

wo $\xi = \frac{1}{2} \sqrt{\frac{C_3 - C_1}{C_3 - C_2}}$ und $U_n(\xi) = \frac{\sin (n+1) \arccos \xi}{\sin \arccos \xi}$ ist.

Aus (4) schliessen wir, dass damit die Koeffizienten reell seien, ξ entweder nur reelle oder rein imaginäre Werte annehmen muss. Ausserdem kann mann sich, selbstverständlich, auf die Betrachtung der Werte $\xi > 0$, und Im $\xi \geqslant 0$ beschränken. Dabei kommen drei Fälle in Betracht:

$$1)\ 0 < \xi < 1; \quad 2)\ \xi \geqslant 1; \quad 3)\ \xi = i\eta \qquad (\eta \geqslant 0).$$

Wir beweisen nun den folgenden

Satz 2. *Damit ein Féjèr-Szegösches Polynomsystem, für das die Bedingung* (4) *erfüllt ist, orthogonal sei, sind folgende Beschränkungen hinreichend:*

$$\left.\begin{aligned} -\xi < \frac{C_1}{2C_2} < 1, \quad & wenn\ \xi \geqslant 1 \\ -\eta^2 < \frac{C_1}{2C_2} < 1, \quad & wenn\ \xi = i\eta \quad (\eta \geqslant 0) \end{aligned}\right\} \tag{5}$$

Wenn $0 < \xi < 1$ *ist, so ist die Orthogonalität überhaupt unmöglich.*

Zum Beweis haben wir nur den expliziten Ausdruck für die λ_n, die in der Rekursionsbeziehung (3) vorkommen, zu betrachten:

$$4\lambda_n = \frac{\left[C_2 + (C_2 - C_1)\frac{U_{n-2}(\xi) + U_1(\xi)}{U_n(\xi)}\right]\left[C_2 + (C_2 - C_1)\frac{U_{n-2}(\xi) - U_1(\xi)}{U_n(\xi)}\right]}{\left[C_2 + (C_2 - C_1)\frac{U_{n-2}(\xi)}{U_n(\xi)}\right]\left[C_2 + (C_2 - C_1)\frac{U_{n-3}(\xi)}{U_{n-1}(\xi)}\right]} \tag{6}$$

Wir wollen noch den folgenden Satz, der aus einem Satze von H. O. Blumenthal folgt, anführen:

Satz 3. *Für ein Féjèr-Szegösches orthogonales Polynomsystem ist das Orthogonalitätsintervall endlich; und zwar ist die Menge der Wachstumstellen der Funktion* $\psi(x)$ *überall dicht im Intervall* $(-1, +1)$. *Im Äusseren des Intervalls* $(-1, +1)$ *kann nur eine abzählbare Menge der Wachstumsstellen vorkommen, deren Häufungsstellen dem Intervall* $(-1, +1)$ *gehören.*

Zum Beweis braucht nur beachtet zu werden, dass aus (6) die Existenz des Grenzwertes $\lim_{n \to \infty} \lambda_n = \frac{1}{4}$ folgt.

Elektrotechnisches Institut.
Kharkow.

Eingegangen
den 24. I. 1941.

ZITIERTE LITERATUR

[1] L. Féjèr, Math. ZS., XXIV, 285 (1926). [2] G. Szegö, ibid., XXV, 172 (1926).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

MATHÉMATIQUES

APPROXIMATION DES FONCTIONS CONTINUES AU MOYEN DE POLYNÔMES DE LA FORME $e^{-nx}\sum_{k=0}^{m} C_{k,n}\,x^k$

Par G. MIRAKYAN

(Présenté par S. N. Bernstein, de l'Académie, le 20. I. 1941)

Soient m et n deux entiers positifs tels que la limite inférieure de $\frac{m}{n}$ pour $n \to \infty$ est positive, c'est-à-dire

$$\lim_{n\to\infty} \frac{m}{n} = r > 0. \tag{1}$$

En choisissant convenablement la croissance de m et n, on peut faire de r un nombre positif quelconque > 0. Prenons un nombre $r' > 0$ de sorte que $r > r' > 0$. Démontrons le

Lemme. *Soit* $0 \leqq x \leqq r'$; *alors*

$$\lim_{n\to\infty} \sum_{k=0}^{m} \frac{n^k x^k}{k!}\, e^{-nx} = 1 \quad \textit{uniformément dans } [0,r']. \tag{2}$$

Choisissons m et n assez grand pour que $\frac{m}{n} > r'$. Il suit de la formule de Taylor pour la fonction e^{nx}, en appliquant la formule de Stirling pour $m!$, que l'on a

$$\sum_{k=0}^{m} \frac{n^k x^k}{k!}\, e^{-nx} =$$

$$= 1 - \frac{\sqrt{n}\, e^{-\frac{\theta}{12n}}}{\sqrt{2\pi \frac{m}{n}}} \cdot \int_0^x \left[\frac{n}{m} u e^{-\frac{n}{m}\left(u-\frac{m}{n}\right)}\right]^m du = 1 - A_{m,n} \qquad (0<\theta<1) \tag{3}$$

où l'on a posé

$$A_{m,n} = \frac{\sqrt{n}\, e^{-\frac{\theta}{12m}}}{\sqrt{2\pi \frac{m}{n}}} \int_0^x \left[\frac{n}{m} u e^{-\frac{n}{m}\left(u-\frac{m}{n}\right)}\right]^m du \qquad (0<\theta<1). \tag{4}$$

La fonction

$$y=\frac{n}{m}xe^{-\frac{n}{m}\left(x-\frac{m}{n}\right)} \tag{5}$$

croît pour $0\leqq x\leqq\frac{m}{n}$ et possède un maximum pour $x=\frac{m}{n}$. Pour $x=\frac{m}{n}$ la valeur de la fonction est égale à 1. Comme $\frac{m}{n}>r'$ et $0\leqq x\leqq r'$ on a

$$0\leqq\frac{n}{m}xe^{-\frac{n}{m}\left(x-\frac{m}{n}\right)}<q<1$$

où q ne dépend ni de m, ni de n, ni de $x\subset[0\leqq x\leqq r']$. Donc

$$0\leqq A_{m,n}\leqq\frac{e^{-\frac{0}{12m}}}{r'\sqrt{2\pi}}\sqrt{m}\,q^m$$

c'est-à-dire

$$\lim_{n\to\infty}A_{m,n}=0\quad\text{uniformément pour}\quad 0\leqq x\leqq r'. \tag{6}$$

Ainsi le lemme est démontré.

Soit $f(x)$ une fonction continue dans le segment $[a, b]$. On peut transformer le segment $[a, b]$ en $[0, r']$ par une transformation linéaire. Considérons donc une fonction $f(x)$ continue dans le segment $[0, r']$. Désignons la borne supérieure de l'ensemble des valeurs de $\frac{m}{n}$ par L (il peut arriver que $L=+\infty$). Prolongeons la fonction $f(x)$ en la définissant aux points rationnels du segment $[r', L]$ de sorte que partout dans $[0, r']$ et aux points rationnels du segment $[r', L]$ on ait l'inégalité

$$|f(x)|<M. \tag{7}$$

D'ailleurs, nous exigeons que la fonction $f(x)$ soit continue à droite au point $x=r'$, c'est-à-dire $|f(x)-f(r')|<\varepsilon$ pour $0<x-r'<\delta$, x étant un nombre rationnel. Dans la suite x désigne un point du segment $[0, r']$.

Considérons la fonction

$$\varphi_{m,n}(x)=\sum_{k=0}^{m}\frac{n^kx^k}{k!}e^{-nx}f\left(\frac{k}{n}\right). \tag{8}$$

Démontrons que $\lim_{n\to\infty}\varphi_{m,n}(x)=f(x)$ uniformément dans $[0, r']$, si $\lim_{n\to\infty}\frac{m}{n}=$ $=r>r'>0$.

D'après le lemme, l'expression (8) donne une approximation pour l'unité. La convergence de $\varphi_{m,n}(x)$ vers $f(x)$ s'établit par une méthode analogue à celle de S. Bernstein [1]. Un exposé du résultat de S. Bernstein peut aussi être trouvé dans la monographie de M. Gontcharoff [2]. Le contenu de la première partie de la présente Note aurait pu, comme il a été fait dans l'article cité de S. Bernstein, être exposé dans la terminologie de la théorie des probabilités.

Pour démontrer que $\varphi_{m,n}(x)$ converge vers $f(x)$, évaluons la différence

$$f(x)-\varphi_{m,n}(x)=f(x)-\sum_{k=0}^{m}\frac{n^kx^k}{k!}e^{-nx}f\left(\frac{k}{n}\right)=$$

$$=\sum_{k=0}^{m}\frac{n^kx^k}{k!}e^{-nx}\left[f(x)-f\left(\frac{k}{n}\right)\right]+A_{m,n}f(x). \tag{9}$$

Soit ensuite ε un nombre quelconque > 0. Alors, en tenant compte de la continuité uniforme de $f(x)$ dans $[0, r']$, prenons un δ indépendant de x et tel que pour $\left|x-\frac{k}{n}\right|<\delta$ on ait $\left|f(x)-f\left(\frac{k}{n}\right)\right|<\varepsilon$.

Ensuite

$$|f(x)-\varphi_{m,n}(x)| \leqq \sum{}' \frac{n^k x^k}{k!} e^{-nx}\left|f(x)-f\left(\frac{k}{n}\right)\right| + \\ + \sum{}'' \frac{n^k x^k}{k!}\left|f(x)-f\left(\frac{k}{n}\right)\right| + A_{m,n}\cdot M \qquad (10)$$

où $\sum'$ s'étend aux valeurs de k pour lesquelles

$$\left|x-\frac{k}{n}\right|<\delta,$$

et $\sum''$ aux autres valeurs, c'est-à-dire celles, pour lesquelles

$$\left|x-\frac{k}{n}\right| \geqq \delta. \qquad (11)$$

Evaluons d'abord la première somme dans le second membre de (10); on a

$$\sum{}' \frac{n^k x^k}{k!} e^{-nx}\left|f(x)-f\left(\frac{k}{n}\right)\right| < \varepsilon \sum{}' \frac{n^k x^k}{k!} e^{-nx} < \varepsilon \sum_{k=0}^{m} \frac{n^k x^k}{k!} e^{-nx} = \\ = \varepsilon (1 - A_{m,n}) \qquad (12)$$

donc $\sum'$ est aussi petit que l'on veut.

Maintenant évaluons la seconde somme du second membre de (10). En tenant compte de (11) et (7), on aura

$$\sum{}'' \frac{n^k x^k}{k!} e^{-nx}\left|f(x)-f\left(\frac{k}{n}\right)\right| < \frac{2M}{n^2\delta^2} \sum{}'' (nx-k)^2 \frac{n^k x^k}{k!} e^{-nx}$$

ou bien, après quelques transformations simples

$$\sum{}'' \frac{n^k x^k}{k!} e^{-nx}\left|f(x)-f\left(\frac{k}{n}\right)\right| < \\ < \frac{2M}{\delta^2}\left[\frac{x}{n}(1-A_{m-1,n}) - x^2(A_{m,n} - 2A_{m-1,n} + A_{m-2,n})\right].$$

Donc, $\sum''$ est aussi petit que l'on veut pour n assez grand en vertu de (6).

Le troisième terme du second membre de (10) est encore aussi petit que l'on veut, si n est assez grand. Ainsi pour chaque $\eta > 0$

$$|f(x)-\varphi_{m,n}(x)| < \eta$$

n est assez grand, et cela uniformément dans $[0, r']$.

Comme la fonction $\varphi_{m,n}(x)$ peut être approchée par un polynôme, par exemple par une somme partielle de son développement de Taylor, avec erreur aussi petite que l'on veut, il en résulte que le même polynôme donne pour la fonction $f(x)$ une approximation uniforme dans $[0, r']$ avec un degré d'exactitude donné, ce qui est le contenu du théorème de Weierstrass. La fonction étudiée

$$\varphi_{m,n}(x) = \sum_{k=0}^{m} \frac{n^k x^k}{k!} e^{-nx} f\left(\frac{k}{n}\right)$$

peut être considérée comme une formule interpolatoire dans le segment $[0, r']$ correspondant aux noeuds d'interpolation $0, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{m}{n}$.

Mais la différence entre cette formule et les formules d'interpolation ordinaire, par exemple celle de Lagrange, consiste en ce que les valeurs de $\varphi_{m,n}(x)$ et de $f(x)$ aux noeuds d'interpolation ne sont qu'approximativement égales.

Cependant, la méthode d'interpolation proposée peut être modifiée sans faire tomber la convergence, de sorte qu'en des points du segment donnés d'avance, où la convergence a lieu (le nombre de ces points est borné), la fonction interpolatoire prenne les mêmes valeurs dans les noeuds que $f(x)$ dans les mêmes points.

Le raisonnement qui suit est tout-à-fait général en ce sens que pour qu'il soit valable il suffit pour chaque $\varepsilon > 0$ de pouvoir trouver une fonction $\Phi(x)$ telle que

$$|f(x)-\Phi(x)|<\varepsilon \quad \text{pour} \quad a\leqq x\leqq b. \tag{I}$$

Ici, en particulier, d'après ce qui a été démontré on peut prendre pour $\Phi(x)$ la fonction $\varphi_{m,n}(x)$. Tout d'abord on peut toujours supposer que $f(x)$ ne s'annulle pas dans $[a, b]$, donc, en tenant compte de la continuité de $f(x)$ dans $[a, b]$

$$M\geqq|f(x)|\geqq\lambda>0. \tag{II}$$

En considérant p comme borné, choisissons les nombres $x_k\ (k=1, 2, \ldots, p)$ de la manière suivante:

$$a\leqq x_1<x_2<\ldots<x_p\leqq b.$$

En vertu de (I) et (II), en prenant $2\varepsilon<\lambda$ on a

$$|\Phi(x)|>\frac{\lambda}{2}. \tag{III}$$

La formule d'interpolation cherchée est de la forme

$$F(x)=\Phi(x)\sum_{k=1}^{p}\frac{f(x_k)}{\Phi(x_k)}\,l_k(x), \tag{IV}$$

Ici les $l_k(x)$ sont les polynômes fondamentaux de la formule d'interpolation de Lagrange

$$l_k(x)=\frac{\omega(x)}{(x-x_k)\,\omega'(x_k)}$$

$$\omega(x)=c(x-x_1)(x-x_2)\ldots(x-x_p) \quad (c \text{ est une constante} \neq 0). \tag{V}$$

Remarquons d'ailleurs que

$$\sum_{k=1}^{p} l_k(x)=1. \tag{VI}$$

Il est evident que la fonction $F(x)$ definie par (IV) verifie les conditions

$$F(x_k)=f(x_k) \qquad (k=1, 2, \ldots, p).$$

Il reste à démontrer que

$$|f(x)-F(x)|<\eta \qquad \text{pour } a\leqq x\leqq b,$$

où η est un nombre >0 quelconque. Evaluons la différence $F(x)-f(x)$. En tenant compte de (VI), nous aurons

$$F(x)-f(x)=\sum_{k=1}^{p}\frac{[\Phi(x)-\Phi(x_k)][f(x_k)-f(x)]+[\Phi(x_k)-\Phi(x)]f(x)}{\Phi(x_k)}\,l_k(x). \tag{VII}$$

En tenant compte de (I) on a

$$f(x)=\Phi(x)+\varepsilon\vartheta(x).$$

lci $0\leqq\vartheta(x)<1$ $(a\leqq x\leqq b)$, d'où

$$f(x_k)=\Phi(x_k)+\varepsilon\vartheta_k, \quad \text{où} \quad 0\leqq\vartheta_k=\vartheta(x_k)<1 \quad (k=1, 2, \ldots, p).$$

En substituant dans (VII) et en tenant compte de (VI), on aura

$$F(x)-f(x)=[\Phi(x)-f(x)]\left[1+\sum_{k=1}^{p}\frac{\Phi(x_k)-f(x_k)}{\Phi(x_k)}l_k(x)\right]+\varepsilon\sum_{k=1}^{p}\frac{\vartheta_k}{\Phi(x_k)}l_k(x).$$

Il est évident que le nombre M dans (II) peut être choisi de sorte que l'on ait

$$|\Phi(x)|<M \qquad \text{pour } a\leqq x\leqq b.$$

Alors il suit de (I), (II) et (III) que

$$|F(x)-f(x)|<\varepsilon\left[1+\frac{4M+2}{\lambda}\sum_{k=1}^{p}|l_k(x)|\right].$$

Comme p est borné, la somme $\sum_{k=1}^{p}|l_k(x)|$ l'est aussi dans le segment $[a, b]$. Donc

$$0<\sum_{k=1}^{p}|l_k(x)|<A.$$

Par conséquent

$$|F(x)-f(x)|<\varepsilon\left[1+\frac{4M+2}{\lambda}A\right]. \qquad \text{(VIII)}$$

Ainsi la différence $F(x)-f(x)$ est aussi petite que l'on veut. En prenant une suite de nombres positifs

$$\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n, \ldots \qquad \lim_{n\to\infty}\varepsilon_n=0$$

nous trouverons une suite

$$\Phi_1(x), \Phi_2(x), \ldots, \Phi_n(x), \ldots$$

et une suite

$$F_1(x), F_2(x), \ldots, F_n(x), \ldots$$

Il suit de (VIII) d'une part

$$\lim_{n\to\infty}F_n(x)=f(x) \text{ uniformément dans } [a, b];$$

et d'autre part

$$F_n(x_k)=f(x_k) \qquad (k=1, 2, \ldots, p).$$

Université d'Etat.
Kharkoff.

Manuscrit reçu
le 24. I. 1941.

LITTÉRATURE CITÉE

[1] С. Бернштейн, Сообщ. Харьк. матем. об-ва, (2), XIII (1912). [2] В. Л. Гончаров, Теория интерполир. и приближения функций, 136 (1934).

Traduit par N. Bary.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

MATHÉMATIQUES

UNE GÉNÉRALISATION D'UN THÉORÈME DE M. J. HADAMARD SUR LA CONVERGENCE DES SÉRIES

Par I. OGUIEWETZKI

(Présenté par S. N. Bernstein, de l'Académie, le 26. I. 1941)

Dans cette Note nous étendons au cas des séries fonctionnelles un théorème connu (1) sur la convergence d'une série numérique $\sum_{i=0}^{\infty} \alpha_i u_i$, quelle que soit la série convergente $\sum_{i=0}^{\infty} u_i$, pourvu que les nombres α_i jouissent de la propriété suivante: si $\Delta\alpha_i = \alpha_i - \alpha_{i+1}$, la série $\sum_{i=0}^{\infty} |\Delta\alpha_i|$ est convergente.

Démontrons le lemme suivant.

Lemme. *Supposons que la série* $\sum_{i=0}^{\infty} |\Delta\alpha_i(x)|$ *converge uniformément et que d'ailleurs les* $\alpha_i(x)$ *sont des fonctions continues dans* $[a, b]$; *alors la suite des fonctions* $\alpha_n(x)$ *est uniformément bornée.*

Cette proposition est une conséquence immédiate de l'identité

$$\alpha_0(x) = \sum_{i=0}^{n-1} [\alpha_i(x) - \alpha_{i+1}(x)] + \alpha_n(x). \tag{1}$$

Théorème. *Pour qu'une suite de fonctions* $\alpha_i(x)$ *continues dans* $[a, b]$ *transforme chaque série uniformément convergente* $\sum_{i=0}^{\infty} u_i(x)$ *en une série* $\sum_{i=0}^{\infty} \alpha_i(x) u_i(x)$ *encore uniformément convergente, il faut et il suffit que la série* $\sum_{i=0}^{\infty} |\Delta\alpha_i(x)|$ *converge uniformément.*

En effet, en appliquant la transformation d'Abel (2) nous trouvons:

$$\sum_{i=m+1}^{n} \alpha_i(x) u_i(x) = \sum_{i=m+1}^{n} [S_i(x) - S(x)] \Delta\alpha_i(x) - [S_m(x) - S(x)] \alpha_{m+1}(x) + [S_n(x) - S(x)] \alpha_{n+1}(x); \tag{2}$$

d'où, en vertu de la convergence uniforme des séries $\sum_{i=0}^{\infty} u_i(x)$ et $\sum_{i=0}^{\infty} |\Delta\alpha_i(x)|$ et de ce que les $\alpha_i(x)$ sont uniformément bornées, on trouve

$$\left| \sum_{i=m+1}^{n} \alpha_i(x)\, u_i(x) \right| < \varepsilon'$$

où $\varepsilon' > 0$ est un nombre positif aussi petit qu'on veut si m est assez grand.

Démontrons la nécessité de la condition. Supposons que $\sum_{i=0}^{\infty} \Delta\alpha_i(x)$ ne converge pas uniformément. Considérons une série spéciale pour laquelle

$$S_i(x) = \begin{cases} \operatorname{sgn} \Delta\alpha_i(x)\, k^{-1} & \text{pour } p_k + 1 < i \leqq p_{k+1};\quad k = 1, 2, 3, \ldots \\ 0 & \text{pour } 0 \leqq i \leqq p_1 + 1;\ i = p_2 + 1,\ p_3 + 1, \ldots \end{cases}$$

Cette série converge uniformément tandis que la série $\sum_{i=0}^{\infty} \alpha_i(x)\, u_i(x)$ ne converge pas uniformément. En effet, de ce que $\sum_{i=0}^{\infty} \Delta\alpha_i(x)$ ne converge pas uniformément, il suit qu'il existe un $\varepsilon > 0$ et une certaine suite de points $x_r \subset [a, b]$, où $r = 1, 2, \ldots$, tels que pour chaque $r = 1, 2, \ldots$ et pour un certain $s = S_r \geqq N_r$ qui lui correspond on a

$$\sum_{i=s}^{s+t} |\Delta\alpha_i(x_r)| > \varepsilon.$$

En supposant que $m + 1 = p_1 + 1 = s$ et $n = p_{k+1}$, nous trouvons de (2):

$$\left| \sum_{i=m+1}^{n} \alpha_i(x_r)\, u_i(x_r) \right| \geqq \sum_{i=p_1+2}^{p_2+1} |\Delta\alpha_i(x)| \frac{1}{1} + \sum_{i=p_2+2}^{p_3+1} |\Delta\alpha_i(x)| \frac{1}{2} +$$

$$+ \ldots + \sum_{p_k+2}^{p_{k+1}} |\Delta\alpha_i(x_r)| \frac{1}{k} - \left| \frac{1}{p_{k+1}} \operatorname{sgn} \Delta\alpha_n(x_r)\, \alpha_{n+1}(x_r) \right|$$

c'est-à-dire

$$\left| \sum_{i=m+1}^{n} \alpha_i(x_r)\, u_i(x_r) \right| > \varepsilon \left(1 + \frac{1}{2} + \ldots + \frac{1}{k}\right) - M > \varepsilon$$

où M est le plus grand des nombres $\left|\frac{\alpha_{k+1}(x_r)}{p_{k+1}}\right|$. Donc, la série $\sum_{i=0}^{\infty} \alpha_i(x)\, u_i(x)$ ne converge pas uniformément.

Remarque. Le théorème démontré reste vrai, si les $\alpha_i(x)$ présentent un ensemble de fonctions qui sont uniformément bornées sur un ensemble M.

Université de Dniepropetrovsk.

Manuscrit reçu le 28. I. 1941.

LITTÉRATURE CITÉE

[1] J. Hadamard, Acta Math., 27 (1903). [2] K. Knopp, Theorie und Anwendung der unendlichen Reihen, 357 (1931).

Traduit par N. Bary.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

MATHEMATIK

ÜBER EINE GRENZWERTAUFGABE DER THEORIE DER BIHARMONISCHEN FUNKTIONEN

Von G. A. BRODSKY

(Vorgelegt von A. N. Kolmogoroff, Mitglied der Akademie, den 14. II. 1941)

Das Ziel der vorliegenden Note ist eine biharmonische und im Kreise beschränkte Funktion zu konstruieren, die keine radialen Grenzwerte auf einer Menge positiven Masses besitzt.

Zu diesem Zweck betrachten wir die harmonische Funktion

$$u(r,\varphi)=\sum_{i=1}^{\infty}2^i r^{2^i}\cos 2^i\varphi$$

und beweisen die folgenden Ungleichungen:

$$|u(r,\varphi)|<\frac{K}{1-r}\ (K \text{ hängt nicht von } r \text{ und } \varphi \text{ ab}) \tag{1}$$

$$\int_0^{2\pi}u^2(r,\varphi)\,d\varphi>\frac{1}{32\pi(1-r)^2}\ \text{für}\ r=1-\frac{1}{2^m},\ m\geq 3\ \text{und ganz.} \tag{2}$$

Wie aus unserer Arbeit (1) folgt, ist die Ungleichung (1) der Ungleichung

$$|f(\varphi+h)+f(\varphi-h)-2f(\varphi)|<Kh \tag{1'}$$

äquivalent, wo

$$f(\varphi)=\sum_{i=1}^{\infty}\frac{\cos 2^i\varphi}{2^i}$$

ist. Wählen wir m so, dass

$$\frac{1}{2^m}\leq h<\frac{1}{2^{m-1}}$$

gilt, so erhalten wir:

$$|f(\varphi+h)+f(\varphi-h)-2f(\varphi)|\leq 2\sum_{i=1}^{m-1}\frac{\sin^2 2^{i-1}h}{2^{i-1}}+2\sum_{i=m}^{\infty}\frac{\sin^2 2^{i-1}h}{2^{i-1}}\leq$$

$$\leq 2h^2\sum_{i=1}^{m-1}2^{i-1}+2\sum_{i=m}^{\infty}\frac{1}{2^{i-1}}=2h^2(2^{m-1}-1)+\frac{1}{2^{m-1}}<4h.$$

Zum Beweis der Ungleichung (2) beachte man die leicht zu erhaltende Ungleichung

$$(a-1)^{2n} > a^{2n} - 2na^{2n-1}$$

wo n eine natürliche Zahl und $a \geqslant 2n$ ist. Somit haben wir:

$$\frac{(1-r^2)}{\pi}\int_0^{2\pi} u^2(r,\varphi)\,d\varphi = (1-r)^2 \sum_{i=1}^{\infty} 2^{2i} r^{2^{i+1}} > (1-r)^2 \sum_{i=1}^{m-2} 4^i r^{2^{i+1}} =$$

$$= \frac{1}{4^m}\sum_{i=1}^{m-2} 4^i \left(\frac{2^m-1}{2^m}\right)^{2^{i+1}} > \frac{1}{4^m}\sum_{i=1}^{m-2} \frac{2^{m2^{i+1}} - 2^{i+1}2^{m(2^{i+1}-1)}}{2^{m2^{i+1}}} =$$

$$= \frac{1}{4^m}\sum_{i=1}^{m-2} 4^i (1-2^{i+1-m}) > \frac{1}{2\cdot 4^m}\sum_{i=1}^{m-2} 4^i > \frac{1}{32}.$$

Wir betrachten nun die biharmonische Funktion

$$F(r,\varphi) = (1-r^2)\,u(r,\varphi).$$

Aus der Ungleichung (1) folgt, dass $F(r,\varphi)$ im Einheitskreise beschränkt ist.

Wir schreiben nun die Identität auf:

$$(1-R^2)\,u(r,\varphi) - \frac{1}{2\pi}\int_0^{2\pi} F(R,0)\,\frac{(R^2-r^2)\,d\theta}{R^2+r^2-2Rr\cos(\theta-\varphi)} \quad (r<R<1). \qquad (*)$$

Würde $F(r,\varphi)$ fast überall radiale Grenzwerte $F(1,\varphi)$ besitzen, so hätten wir, in (*) zum Grenzwert $R \to 1$ übergehend,

$$0 = \frac{1}{2\pi}\int_0^{2\pi} F(1,\theta)\,\frac{1-r^2}{1+r^2-2r\cos(\theta-\varphi)}\,d\theta$$

woraus folgen würde, dass $F(1,\theta) = 0$ für fast alle θ ist. Dies ist aber wegen der Ungleichung (2) unmöglich.

Folglich genügt $F(r,\varphi)$ allen anfangs gestellten Bedingungen.

Eingegangen
den 31. I. 1941.

ZITIERTE LITERATUR

[1] Г. А. Бродский, Уч. Зап. МГУ (1941).

Übersetzt von V. Levin.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

MATHEMATICS

ESTIMATIONS OF THE REMAINDER OF FÉJÈR'S SUM FOR PERIODICAL FUNCTIONS POSSESSING A BOUNDED DERIVATIVE

By S. M. NIKOLSKY

(Communicated by A. N. Kolmogoroff, Member of the Academy, 26. I. 1941)

1. Let a function f belonging to a certain class $\mathfrak{M}$ of continuous functions be approximated by a method of approximation (operator) $U_n(n=1, 2, \ldots)$. Denote by $U_n(f, x)$ the value of a corresponding approximation at the point x, and by

$$\mathcal{E}_n(x) = \mathcal{E}_{U_n}(\mathfrak{M}, x) = \sup_{f \subset \mathfrak{M}} |f(x) - U_n(f, x)|$$

the upper bound of the absolute values of differences between the functions f and their approximations at this point, taken over $\mathfrak{M}$.

2. Denote further by $KW^{(r)}$ [$1\,W^{(r)} = W^{(r)}$] the class of functions of period 2π possessing an absolutely continuous $(r-1)$-th derivative and almost everywhere the r-th derivative, satisfying at the points of its existence the inequality $|f^{(r)}(t)| \leqslant K$*.

In the present note we give an asymptotic expression for $\mathcal{E}_n(x)$ for the functions from $KW^{(r)}$ approximated by the Féjèr sums $\sigma_n(f, x)$ and by the interpolated Féjèr sums $\tilde{\sigma}_n(f, x)$ of the n-th order. Hereby we understand that the interpolated Féjèr sums of the n-th degree are trigonometric polynomials of the form

$$\tilde{\sigma}_n(f, x) = \frac{1}{n+1} \sum_{m=0}^{n} \tilde{s}_m^{(n)} \tag{1}$$

where

$$\left.\begin{aligned} &\tilde{s}_0^n = \frac{a_0^{(n)}}{2}, \quad \tilde{s}_m^{(n)} = \frac{a_0^{(n)}}{2} + \sum_{k=1}^{m} (a_k^{(n)} \cos kx + b_k^{(n)} \sin kx), \\ &\qquad (m = 1, 2, \ldots, n) \\ &a_k^{(n)} = \frac{2}{2n+1} \sum_{i=0}^{2n} f(x_i^{(n)}) \cos kx_i^{(n)}, \quad b_k^{(n)} = \frac{2}{2n+1} \sum_{i=0}^{2n} f(x_i^{(n)}) \sin kx_i^{(n)}, \\ &\qquad (k = 0, 1, \ldots, n) \\ &x_i^{(n)} = \frac{2\pi i}{2n+1} \qquad (i = 0, \pm 1, \pm 2, \ldots). \end{aligned}\right\} \tag{2}$$

* This class may also be defined as the set of functions of period 2π possessing the $(r-1)$-st derivative satisfying the Lipschitz condition with the constant K.

We prove the following theorems:

Theorem I. *For* $r=1$ *we have:*

$$\mathcal{E}_{\sigma_n}(KW^{(1)}, x)=\frac{2K}{\pi}\frac{\ln n}{n}+O(n^{-1}); \tag{3}$$

for $r=2, 3, \ldots$

$$\mathcal{E}_{\sigma_n}(KW^{(r)}, x)=\frac{4K}{\pi(n+1)}c_r+O\left(\frac{\ln n}{n^r}\right) \tag{4}$$

where

$$c_r=\sum_{\nu=0}^{\infty}\frac{(-1)^{\nu(r-1)}}{(2\nu+1)^r}.$$

Theorem II. *The equalities* (3) *and* (4) *remain valid when* σ_n *is replaced by* $\tilde{\sigma}_n$.

We shall not dwell upon the proof of equality (3), for it has already been established in a different way in my note [(1)], formulae (2) and (4)]. The corresponding result, however, in case of the method of approximation σ_n could be as well obtained by considerations used here.

3. Proof of Theorem I ($r>1$). In the theorem it is evidently sufficient to consider the class $W^{(r)}$; the obtained result is then to be multiplied by K.

Developing the function $f(x)$ from $W^{(r)}$ into Fourier series and integrating its Fourier coefficients by parts, we obtain:

$$f(x)=\frac{a_0}{2}+\sum_{k=1}^{\infty}\frac{1}{\pi k^r}\int_0^{2\pi}\cos\left[k(t-x)+\frac{r\pi}{2}\right]\varphi(t)\,dt \tag{5}$$

where $f^{(r)}(t)=\varphi(t)$, and we have

$$\int_0^{2\pi}\varphi(t)\,dt=0. \tag{6}$$

Conversely, when a measurable function φ, having the period 2π and not exceeding 1 in absolute value satisfies the relation (5), then the function $f(x)$ defined by the equation (5) belongs to $W^{(r)}$ and its derivative coincides almost everywhere with φ. From (5) follows

$$\sigma_n(f, x)=\frac{a_0}{2}+\sum_{k=1}^{n}\frac{1}{\pi k^2}\frac{n+1-k}{n+1}\int_0^{2\pi}\cos\left[k(t-x)+\frac{r\pi}{2}\right]\varphi(t)\,dt \tag{7}$$

and since in this case $\mathcal{E}_n(x)$ is independent of x, we shall have, assuming that $f(0)$ and $\sigma_n(f, 0)$ are expressed through φ by means of formulae (5) and (7), that

$$\mathcal{E}_n(x)=\mathcal{E}_n(0)=\sup|f(0)-\sigma_n(f, 0)|,$$

where the exact upper bound is extended to all functions φ of the period 2π not exceeding 1 in absolute value and satisfying (6).

Further from (5) and (7) follows the relation

$$f(0)-\sigma_n(f, 0)=\frac{1}{\pi(n+1)}\int_0^{2\pi}F_r(t)\varphi(t)\,dt+$$

$$+\frac{1}{\pi}\int_0^{2\pi}\left\{D_n^{(r)}(t)+\frac{1}{n+1}\overline{D_n^{(r-1)}}(t)\right\}\varphi(t)\,dt \tag{8}$$

where

$$F_r(t) = \sum_{k=1}^{\infty} \frac{1}{k^{r-1}} \cos\left(kt + \frac{r\pi}{2}\right), \quad D_n^{(r)}(t) = \sum_{k=n+1}^{\infty} \frac{1}{k^r} \cos\left(kt + \frac{r\pi}{2}\right),$$

$$\overline{D_n^{(r-1)}}(t) = -\sum_{k=n+1}^{\infty} \frac{1}{k^{r-1}} \cos\left(kt + \frac{r\pi}{2}\right) = \sum_{k=n+1}^{\infty} \frac{1}{k^{r-1}} \sin\left(kt + \frac{r\pi}{2}\right).$$

A. Kolmogoroff has shown (2), that

$$\int_0^{2\pi} |D_n^{(r)}(t)|\, dt = O\left(\frac{\lg n}{n^r}\right). \tag{9}$$

Analogously it may be proved that

$$\int_0^{2\pi} |\overline{D_n^{(r)}}(t)|\, dt = O\left(\frac{\lg n}{n^r}\right). \tag{10}$$

Therefore it follows from (8), that

$$f(0) - \sigma_n(f, 0) = \frac{1}{\pi(n+1)} \int_0^{2\pi} F_r(t)\, \varphi(t)\, dt + O\left(\frac{\lg n}{n^r}\right) \tag{11}$$

uniformly with respect to all functions φ not exceeding 1 in absolute value. Taking into account, that

$$F_2(t) = \lg\left(2 \sin\frac{t}{2}\right) \qquad 0 < t \leqslant \pi$$

$$F'_{r+1}(t) = -F_r(t) \qquad (r = 2, 3, \ldots)$$

we conclude inductively that for an odd r function $F_r(t)$ vanishes only at the points $t = k\pi$ with a change of sign, and that for an even r function $F_r(t) - F_r\left(\frac{\pi}{2}\right)$ vanishes only at the points $t = \frac{\pi}{2} + k\pi$ $(k = 0, \pm 1, \pm 2, \ldots)$.

Then from (11) follows for an odd r and $f \subset W^{(r)}$ that

$$|f(0) - \sigma_n(f, 0)| \leqslant \frac{1}{\pi(n+1)} \int_0^{2\pi} |F_r(t)|\, dt + O\left(\frac{\lg n}{n^r}\right) =$$

$$= \frac{2}{\pi(n+1)} \left| \int_0^{\pi} F_r(t)\, dt \right| + O\left(\frac{\lg n}{n^r}\right) = \frac{4}{\pi(n+1)} c_r + O\left(\frac{\lg n}{n^r}\right),$$

where the sign of equality is attained when $f^{(r)}(t) = \varphi(t) = \operatorname{sign} \sin t$.

If r is even, then, in view of (6) we can write (11) in the form

$$f(0) - \sigma_n(f, 0) = \frac{1}{\pi(n+1)} \int_0^{2\pi} \left[F_r(t) - F\left(\frac{\pi}{2}\right)\right] \varphi(t)\, dt + O\left(\frac{\lg n}{n^r}\right)$$

whence

$$|f(0)-\sigma_n(f,0)| \leqslant \frac{1}{\pi(n+1)} \int_0^{2\pi} \left| F_r(t) - F\left(\frac{\pi}{2}\right) \right| dt + O\left(\frac{\lg n}{n^r}\right) =$$

$$= \frac{2}{\pi(n+1)} \left| \int_0^{\frac{\pi}{2}} \left[F_r(t) - F\left(\frac{\pi}{2}\right) \right] dt - \int_{\frac{\pi}{2}}^{\pi} \left[F_r(t) - F\left(\frac{\pi}{2}\right) \right] dt \right| +$$

$$+ O\left(\frac{\lg n}{n^r}\right) = \frac{4}{\pi(n+1)} c_r + O\left(\frac{\lg n}{n^r}\right);$$

and the sign of equality is attained when $f^{(r)}(t) = \varphi(t) = \operatorname{sign} \cos t$. This completes the proof.

4. Theorem II is for $r > 1$ obviously a consequence of the formula (4) and the following lemma:

Lemma 1. *If $r > 1$, we have:*

$$\Delta_n^{(r)} = \sup_x \sup_{f \subset W^{(r)}} |\tilde{\sigma}_n(f,x) - \sigma_n(f,x)| = O\left(\frac{\lg n}{n^r}\right).$$

The proof of the lemma runs as follows. Using the well-known formulae [see for example (3) or (4)]

$$a_k^{(n)} = a_k + \sum_{\mu=1}^{\infty} (a_{\mu(2n+1)+k} + a_{\mu(2n+1)-k}) \qquad (k = 0, 1, \ldots)$$

$$b_k^{(n)} = b_k + \sum_{\mu=1}^{\infty} (b_{\mu(2n+1)+k} - b_{\mu(2n+1)-k}) \qquad (k = 1, 2, \ldots)$$

where a_ν and b_ν are the Fourier coefficients of f, we shall have from (1) and (2):

$$\tilde{\sigma}_n(f,x) - \sigma_n(f,x) = \sum_{\mu=1}^{\infty} a_{\mu(2n+1)} +$$

$$+ \sum_{k=1}^{n} \frac{n+1-k}{n+1} \left\{ \sum_{\mu=1}^{\infty} (a_{\mu(2n+1)+k} + a_{\mu(2n+1)-k}) \cos kx + \sum_{\mu=1}^{\infty} (b_{\mu(2n+1)+k} - \right.$$

$$\left. - b_{\mu(2n+1)-k}) \sin kx \right\}; \qquad (12)$$

$$a_\nu = \frac{1}{\pi^r} \int_0^{2\pi} \cos\left(\nu t + \frac{r\pi}{2}\right) \varphi(t)\, dt, \qquad b_\nu = \frac{1}{\pi^r} \int_0^{2\pi} \sin\left(\nu t + \frac{r\pi}{2}\right) \varphi(t)\, dt$$

$$(\nu = 1, 2 \ldots).$$

Hence, combining those of the terms of the sum on the right-hand side of (12), whose Fourier coefficients for f have the same indices, and defining the functions $\omega_\nu (\nu = n+1, n+2, \ldots)$ which depend on t and x by the formulae

$$\omega_{\mu(2n+1)+k} = \frac{n+1-k}{n+1} \cos\left[\mu(2n+1)t + k(t-x) + \frac{r\pi}{2}\right]$$

$$\omega_{\mu(2n+1)-k} = \frac{n+1-k}{n+1} \cos\left[\mu(2n+1)t - k(t-x) + \frac{r\pi}{2}\right]$$

$$(\mu = 1, 2, \ldots, \quad k = 0, 1, \ldots)$$

we obtain

$$\tilde{\sigma}_n(f,x)-\sigma_n(f,x)=\frac{1}{\pi}\int_0^{2\pi}\sum_{\nu=n+1}^{\infty}\frac{\omega_\nu}{\nu^r}\varphi(t)\,dt. \tag{13}$$

Further we can prove the inequality

$$\frac{1}{\pi}\int_0^{2\pi}\Big|\sum_{\nu=n+1}^{l}\omega_\nu\Big|\,dt\leqslant c\lg l \qquad (l=n+1,\ n+2,\ \ldots)$$

where the constant c is independent of l, n and x, which proves the lemma, when we apply Abel's transformation to the right-hand side of (13).

5. Note. For any positive number r (not necessarily an integer) a class of functions $KW^{(r)}$ may be considered, being a set of functions of the period 2π having almost everywhere a r-th derivative $f^{(r)}(t)=\varphi(t)$ in the sense of Weyl not exceeding a constant K in absolute value.

We speak of a function f of period 2π as having a summable function $\varphi(t)$ of period 2π for its r-th derivative $(r>0)$ in the sense of Weyl, if the equations (5) and (6) hold. In such a case are valid the relations

$$O(n^{-r}) \qquad 0<r<1 \tag{14}$$

$$\mathcal{E}_{\sigma_n}(KW^{(r)},\ x)=O\Big(\frac{\lg n}{n^r}\Big) \qquad r=1 \tag{14'}$$

$$O(n^{-1}) \qquad r>1 \tag{14''}$$

which hold good when σ_n is replaced by $\tilde{\sigma}_n$. Hereby the sign O cannot be replaced by o anywhere. The relation (14) is a consequence of equations (2) and (4) from [1] [see also (5) for σ_n] and the following theorem due to Hardy and Littlewood [6] which can be formulated as follows.

If a function f of period 2π has an r-th derivative $(0<r\leqslant 1)$ in sense of Weyl which is bounded by a constant K in absolute value, then f satisfies a Lipschitz condition of degree r:

$$|f(x')-f(x'')|\leqslant M|x'-x''|^r.$$

where M depends only on K and r.

The equation (14') is a consequence of (3), while (14'') is a consequence of (11), which is proved in this general case in the same manner as for r integer [7]. The Lemma 1 holds for any $r>0$.

Steklov Institute of Mathematics.
Academy of Sciences of the USSR.
Moscow.

Received
28. I. 1941.

REFERENCES

[1] S. Nikolsky, Bull. Acad. Sci. URSS, 4, No. 6 (1940). [2] A. Kolmogoroff, Ann. of Math., 36, No. 2 (1935). [3] D. Jackson, The Theory of Approximation, N. Y. (1930). [4] J. Marcinkiewiez, Studia Math., VI (1936). [5] A. Zigmund, Trigonometrical series, 3, 68 (1939). [6] G. H. Hardy and I. E. Littlewood, Mathem. ZS., 27, 589 (1928). [7] W. Pinkewitch, Bull. Acad. Sci. URSS, 4, No. 6 (1940).

Translated by V. Bermann.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

MATHEMATICS

AN ASYMPTOTIC ESTIMATION OF THE REMAINDER UNDER APPROXIMATION BY INTERPOLATING TRIGONOMETRIC POLYNOMIALS

By S. M. NIKOLSKY

(Communicated by A. N. Kolmogoroff, Member of the Academy, 28. I. 1941)

1. An interpolation trigonometric polynomial of the n-th degree coinciding with a function f of period 2π at the points

$$x_k^{(n)} = kh, \qquad h = \frac{2\pi}{2n+1} \qquad (k = 0, \pm 1, \pm 2, \ldots)$$

is expressed, as it is known, by the formula

$$\tilde{s}_n | f, x | = \frac{2}{2n+1} \sum_{k=0}^{2n} D_n (x - x_k^{(n)}) f(x_k^{(n)}) \tag{1}$$

where

$$D_n(u) = \frac{\sin\left(n+\frac{1}{2}\right)u}{2\sin\frac{u}{2}} \qquad (n = 1, 2, \ldots).$$

According to the preceding note (1), we use the notations

$$\mathcal{E}_n(x) = \mathcal{E}_{u_n}(\mathfrak{M}, x) = \sup_{f \subset \mathfrak{M}} | f(x) - U_n(f, x) |$$

where $\mathfrak{M}$ is a class of functions and U_n a certain method of approximation.

Let $KW^{(r)}$ [$1W^{(r)} = W^{(r)}$] again denote the class of functions of period 2π having an absolutely continuous $(r-1)$-th derivative and almost everywhere a r-th derivative not exceeding a constant K in absolute value ($r = 1, 2, \ldots$). Let further $KH^{(\alpha)}$ [$1H^{(\alpha)} = H^{(\alpha)}$] denote the class of functions of period 2π satisfying a Lipschitz condition of degree α ($0 < \alpha \leqslant 1$) with a constant K: for any two values x' and x'' holds the inequality

$$| f(x') - f(x'') | \leqslant K | x' - x'' |.$$

In the present note we prove the following theorems.

Theorem I. *The equality*

$$\mathcal{E}_n(x) = \mathcal{E}_{\tilde{s}_n}(KH^{(\alpha)}, x) = \frac{K \lg n}{\pi^{1-\alpha} n^{\alpha}} \left| \sin\left(n+\frac{1}{2}\right)x \right| + O(n^{-\alpha}) \tag{2}$$

holds uniformly with respect to x.

Theorem II. *The equality*

$$\mathcal{E}_n(x) = \mathcal{E}_{\tilde{s}_n}(KW^{(r)}, x) = F_n^{(r)} M_n(x) K + O(n^{-r}) \tag{3}$$

holds uniformly with respect to x, where

$$M_n(x) = \frac{2}{2n+1} \sum_{k=0}^{2n} |D_n(x - x_k^{(n)})|$$

is the norm of the linear functional (1) *and* $F_n^{(r)}$ *is the upper bound* $F_n^{(r)} = \sup_{f \subset W^{(r)}} E_n(f)$ *of deviations* $E_n(f)$ *of the functions* f *from their best trigonometric polynomials of the* n*-th degree, this upper bound being extended to the class* $W^{(r)}$.

Owing to the investigations of J. Favard ([2]) and those of N. Achyeser and M. Krein ([3]), this constant becomes known to be equal to

$$F_n^{(r)} = \frac{4}{\pi (n+1)^r} A_r$$

where

$$A_r = \sum_{\nu=0}^{\infty} \frac{(-1)^{\nu(r-1)}}{(2\nu+1)^{r+1}} \qquad (r = 1, 2, \ldots)$$

and it may be obtained as the maximum of the absolute value of the function $f_{nr}(t)$ defined by the equations

$$f_{n0}(t) = \operatorname{sign} \sin (n+1)\, t$$

$$f_{n,r+1}(t) = \int_0^t f_{nr}(u)\, du \qquad \text{for } r = 2p+1$$

$$f_{n,r+1}(t) = \int_{\frac{\pi}{2(n+1)}}^{t} f_{nr}(u)\, du \qquad \text{for } r = 2p$$

(p — an integer). It is known as well that [([4]), theorem III for $p=0$]

$$M_n(x) = \frac{2}{\pi} \lg n \left| \sin \left(n + \frac{1}{2} \right) x \right| + O(1)$$

where $O(1)$ is a function uniformly bounded with respect to x and $n = 1, 2, \ldots$.

Therefore (3) *can be written also in the form*

$$\mathcal{E}_{\tilde{S}_n}(KW^{(r)}, x) = \frac{8}{\pi^2} A_r \frac{K \lg n}{n^r} \left| \sin \left(n + \frac{1}{2} \right) x \right| + O(n^{-r})$$

uniformly with respect to x.

Proving either of the theorems it is sufficient, as can easily be seen, to consider classes $H^{(r)}$ and $W^{(r)}$, respectively; the results can thereupon be multiplied by K. Since in both cases functions $\mathcal{E}_n(x)$ have the period h and for $x = 0$ the theorems are trivial, it is sufficient to consider the case $0 < x < h$.

2. The proof of the Theorem I is essentially based on the following properties of the function $D_n(u)$ for $0 < x < h$

$$\operatorname{sign} d_k = \operatorname{sign} D_n(x - x_k^{(n)}) = (-1)^{k+1} \qquad (k = 1, 2, \ldots, l)$$

$$\operatorname{sign} d_k = \operatorname{sign} D_n(x - x_k^{(n)}) = (-1)^k \qquad (k = 0, -1, \ldots, -l_1)$$

and l and l_1 are the greatest natural numbers satisfying respectively the inequalities $x - \pi \leqslant x_{l_1}^{(n)}$, $x_l^{(n)} < x + \pi$, and

$$d_k = \sum_{\nu=k}^{l} D_n(x - x_\nu^{(n)}) \qquad (k = 1, 2, \ldots l)$$

$$d_k = \sum_{\nu=-l_1}^{k} D_n(x - x_\nu^{(n)}) \qquad (k = 0, -1, \ldots, -l_1).$$

Without loss of generality, we may confine ourselves to the consideration of functions $f(t) \subset H^{(\alpha)}$ vanishing at $t = x$; further taking into account that because of the periodicity of the functions $D_n(u)$ and f the summation on the right-hand part of (1) may be extended over all indices k, for which $-l_1 \leqslant k \leqslant l$, we shall have by Abel's transformation:

$$\tilde{s}_n(f, x) - f(x) = \frac{2}{2n+1}\left\{f(x_1^{(n)})\, d_1 + f(x_0^{(n)})\, d_0 + \right.$$
$$\left. + \sum_{k=2}^{l} [f(x_k^{(n)}) - f(x_{k-1}^{(n)})]\, d_k + \sum_{k-1}^{l_1} [f(x_k^{(n)}) - f(x_{-(k-1)}^{(n)})]\, d_{-k}\right\}$$

whence

$$|\tilde{s}_n(f, x)| \leqslant \left(\frac{2}{2n+1}\right)^{\alpha+1} \pi^\alpha \sum_{k=-l_1}^{k=l} |d_k| + O(n^{-\alpha}) \tag{4}$$

functions d_k being bounded by a constant which is independent of k, n and x.

On the other hand, the function $f_n(t)$ of period 2π which is defined by equations

$$\begin{array}{ll}
0 & 0 \leqslant t \leqslant x_1^{(n)} \\
-\dfrac{(t - x_1^{(n)})^\alpha}{2} & x_1^{(n)} \leqslant t \leqslant x_2^{(n)} \\
-\dfrac{(-t)^\alpha}{2} & x_{-1}^{(n)} \leqslant t \leqslant 0 \\
(-1)^{k+1}\left[\dfrac{h^\alpha}{2} - (t - x_k^{(n)})^\alpha\right] & x_k^{(n)} \leqslant t \leqslant x_{k+1}^{(n)} \qquad (k = 2, 3, \ldots, l) \\
(-1)^k\left[\dfrac{h^\alpha}{2} - (t - x_k^{(n)})^\alpha\right] & x_k^{(n)} \leqslant t \leqslant x_{k+1}^{(n)} \qquad (k = -2, -3, \ldots, -l_1)
\end{array}$$

belongs to the considered class and $\tilde{s}_n(f_n, x)$ is equal to the right-hand part of inequality (4) and, consequently,

$$\mathcal{E}_{\tilde{s}_n}(H^{(\alpha)}, x) = \tilde{s}_n(f_n, x) + O(n^{-\alpha}).$$

Estimating this expression, we obtain (2).

3. The proof of the Theorem II is based on

Lemma. *There exists a positive constant L depending only on r, such that for any sequence of numbers $\gamma_0, \gamma_1, \ldots, \gamma_{r-1}$ in the interval $a \leqslant t \leqslant a + \omega = b$ (or $b = a - \omega \leqslant t \leqslant a$) there can be constructed a function $\Phi(t)$ having a continuous r-th derivative satisfying the following properties:*

$$\Phi^{(k)}(a) = 0 \quad \Phi^{(k)}(b) = \gamma_{r-1-k} \quad (k = 0, 1, \ldots, (r-1))$$
$$\Phi^{(0)}(a) = \Phi(a), \quad |\Phi^{(r)}(t)| \leqslant \sup_k \frac{|\gamma_k|}{\omega^{k+1}} L.$$

Further we proceed the following way. On the one hand, because of S. N. Bernstein's inequality(5) we have

$$|f(x) - s_n(f, x)| \leqslant [1 + M_n(x)]\, E_n(f),$$

whence for $W^{(r)}$ holds

$$\mathcal{E}_{\tilde{s}_n}(W^{(r)}, x) = F_n^{(r)} M_n(x) + O(n^{-r}).$$

On the other hand, we prove the possibility of constructing a function $\varphi_n(t)$ belonging to $W^{(r)}$ for which the value $|\varphi_n(x)-\tilde{s}_n(\varphi_n x)|$ equals the right-hand side of the latter inequality.

We define this function within the interval $x \leqslant t \leqslant x+2\pi$ for $0<x<h$ as follows. Let k_n and k_n^* be the natural numbers satisfying the inequality $1 \leqslant k_n < k_n^* \leqslant 2n+1$; to the more exact definition of k_n and k_n^* we shall proceed later. We put

$$\varphi_n(t) = \bar{f}_{nr}(t)$$

within the interval $k_n h \leqslant t \leqslant k_n^* h$, where

$$\bar{f}_{nr}(t) = f_{nr}\left(\frac{2n+1}{2(n+1)}\,t\right)(-1)^p \quad \text{for } r = 2p+1$$

$$\bar{f}_{nr}(t) = f_{nr}\left(\frac{\overline{2n+1}\,t+\pi}{2(n+1)}\right)(-1)^{p+1} \quad \text{for } r = 2p$$

(p — an integer).

Owing to such a definition, the function $\varphi_n(t)$ has a $(r-1)$-th continuous derivative in the mentioned interval and, except a finite number of points, a r-th continuous derivative not exceeding 1 in absolute value, which takes at the points $x_k^{(n)}$ of this interval the values $\pm F_n^{(r)}$ with the signs coinciding with those of sign $D_n(x-x_k^{(n)})$, respectively.

Translate now the function $\varphi_n(t)$ over the interval $x \leqslant t \leqslant x+2\pi$ so that it should have there continuous derivatives up to the $(r-1)$-th order inclusively, vanishing at $t=x$ and $t=x+2\pi$, and that the r-th derivative should be continuous and not exceed 1 in absolute value in the intervals $x \leqslant t \leqslant k_n h$ and $k_n^* h \leqslant t \leqslant 2\pi + x$. Such a translation is possible because of the mentioned lemma and the properties of functions $f_{nr}(t)$ under a special choice of the numbers k_n and k_n^* which satisfy, however, the inequalities $k_n < c$, $2n+1-k_n^* < c$, where c is independent of n.

As a consequence of the latter inequalities we have that in computation

$$\tilde{s}_n(\varphi_n,\ x) - \varphi_n(x) = \tilde{s}_n(\varphi_n,\ x) = \frac{2}{2n+1}\sum_{k=1}^{2n+1} D_n(x-x_k^{(n)})\ \varphi_n(x_k^{(n)})$$

the numbers

$$\varphi_n(x_k^{(n)}) \quad (k=1, 2, \ldots, k_n \text{ and } k=k_n^*,\ (k_n^*+1), \ldots, (2n+1))$$

can be replaced by the numbers $\bar{f}_{nr}(x_k^{(n)})$, respectively, whence

$$\tilde{s}_n(\varphi_n,\ x) = F_n^{(r)}\,\frac{2}{2n+1}\sum_{k=1}^{2n+1} |D_n(x-x_k^{(n)})| + O(n^{-r}) =$$

$$= F_n^{(r)} M_n(x) + O(n^{-r}).$$

Steklov Institute of Mathematics.
Academy of Sciences of the USSR. Moscow.

Received
28. I. 1941.

REFERENCES

[1] S. M. Nikolsky, C. R. Acad. Sci. URSS, this issue. [2] J. Favard, Bull. des Sciences Math., 209 (1937). [3] N. Achyeser and M. Krein, C. R. Acad. Sci. URSS, XV, No. 3 (1937). [4] S. Nikolsky, Bull. Acad. Sci. URSS, 4, No. 6 (1940). [5] A. Kolmogoroff, Rec. Math., 41, 99 (1934).

Translated by V. Bermann.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

PHYSICS

STRUCTURE OF THE RAYLEIGH LINE AND VISCOSITY OF LIQUIDS

By E. GROSS and A. SIROMYATNIKOV

(Communicated by S. I. Wawilow, Member of the Academy, 22. I. 1941)

As shown by experiments (1) monochromatic light ceases to be monochromatic when scattered in a liquid. A spectrogram of light scattered by a liquid, if obtained with an apparatus of high resolving power, would show, in addition to the frequency of the exciting radiation, also a radiation of an altered wave-length. On such a spectrogram, the Rayleigh line would appear as a triplet with an unmodified line in the middle.

The presence of displaced components of the Rayleigh line had been predicted by the theory of Mandelstam (2) and by that of Brillouin (3). The altered frequency of the exciting light is due, in this case, to density fluctuations, which vary rapidly in time. These rapidly changing fluctuations of density may be represented, according to Debye's (4) conceptions, as a result of the superposition of elastic heat waves propagating in the liquid at sound velocity.

In addition to the displaced components of the Rayleigh line, however, there appears an undisplaced line on the spectrogram of the light scattered by liquids. As shown by Landau and Placzek (5), this line is due to density fluctuations varying slowly in time. Landau and Placzek's theory gives the following expression for the intensity ratio of the displaced and the undisplaced components of the Rayleigh line: $\frac{C_v}{2} : C_p - C_v : \frac{C_v}{2}$.

As shown by the experiments carried out by Birus (6) with the special purpose of checking this ratio, it is well justified for scattering in toluol and water. As may further be seen from the experiments (1) of one of the authors of the present communication, a number of other liquids show an intensity ratio of the same order between the components of the Rayleigh line as that required by the Landau and Placzek theory.

Such an intensity ratio, however, does not hold good for all the liquids. Highly viscous liquids show a very strong undisplaced line. This has been observed by one of us (*E. G.*) in benzyl alcohol, which has a rather high viscosity ($\eta = 0.056$). Similar observations have recently been made by Raman and co-workers (7) for glycerine and phenol.

If the liquid is viscous enough, the intensity of the undisplaced component, as compared to that of the displaced components, is many times as high as it ought to be according to the Landau and Placzek ratio.

The ratio of the intensities of light scattered by rapidly and slowly varying fluctuations of density, respectively, does not hold good for viscous liquids, owing to other supplementary causes which give rise to an additional scattering of light having an unmodified (or slightly modified) wave-length.

In such viscous liquids whose molecules show a high optical anisotropy, the enhanced intensity of the undisplaced line must be due to the light scattered by the fluctuations in the orientation of the molecules in the liquid. This follows from the considerations ([8]) suggested by one of the authors, who had connected the scattering of light due to fluctuations in the orientation of the molecules with relaxation phenomena in the liquid.

In the case of liquids having anisotropic molecules, the spectrogram of scattered light shows, in the neighbourhood of the Rayleigh line triplet, a continuous depolarized background beginning from the undisplaced component of the triplet. The extension of the background over 10—20 cm^{-1} outwards from the Rayleigh line may be observed with the help of a prism spectrograph. One of the writers of the present paper has suggested ([8]) an explanation for this phenomenon.

Owing to thermal motion, the molecules of the liquid perform, in addition to translational vibrations, also angular oscillations about their centres of gravity. These angular oscillations of the molecules taking place during the process of light scattering produce changes in the frequency of the exciting light. A liquid exhibits a whole spectrum of frequencies of those angular oscillations of the molecules. It is this spectrum that gives rise to changes in the frequency of the exciting light resulting in the appearance of a continuous depolarized background in the neighbourhood of the undisplaced component of the Rayleigh line. The extension of that background on both sides of the Rayleigh line is determined by relaxation phenomena taking place in the liquid and is connected with Debye's relaxation time:

$$\tau = \frac{8\pi\eta a^3}{2kT}.$$

The intensity of the continuous background becomes small at that point of the spectrogram of scattered light which corresponds to the frequencies of the angular oscillations of the molecules of the order of $\frac{1}{\tau}$. Therefore, the greater the viscosity of the liqud, i. e. the greater the time of relaxation of the molecules, the shorter the extension of the background on either side of the undisplaced component of the Rayleigh line. In the case of highly viscous liquids, no Rayleigh line background is to be observed. In a highly viscous liquid, the molecules can only perform very slow angular oscillations, and therefore the change in the frequency of the exciting light is too slight to be revealed by a spectral apparatus. Thus, in the case of highly viscous liquids practically the whole intensity of the depolarized light, scattered owing to the orientation fluctuations of the molecules, must be centered in the depolarized undisplaced component of the Rayleigh line.

These conceptions lead to the following inference. In viscous liquids there ought to be a connexion between the intensity of the depolarized undisplaced component of the Rayleigh line and the depolarized background which is observed near the components of the triplet. A decrease in viscosity (reduction in the relaxation time of the liquid) must be followed by a decrease in the number of molecules performing slow angular oscillations, and by an increase in the number of those oscillating at a higher rate. Therefore, as the viscosity of a liquid is decreased, the intensity of the undisplaced line must diminish, and a depolarized continuous background is to appear in its neighbourhood.

We have carried out a number of new experiments in order to check the validity of the above considerations. A Michelson echelon grating was used to investigate the spectrum of scattering of two viscous liquids, *p*-cresol and phenol, at varying temperatures. Those two liquids have been selected as objects of investigation, for their molecules, in addition to their rather

high viscosity, possess a high optical anisotropy. The experiments were, therefore, likely to yield unambiguous results. In order to facilitate the investigation by eliminating from the spectrogram the displaced components of the triplet due to density fluctuations (and in that particular case being of no interest to us), the depolarized portion of the scattered light was investigated. In front of the slit of the spectrograph was placed a Glan Thompson's polarizing prism which transmitted only the light having its electric vector vibrations parallel to the direction of the exciting beam of light. Since the components of the triplet due to density fluctuations are strongly polarized in the direction normal to the exciting beam, they were considerably weakened by the polarizing prism and did not interfere with the investigation.

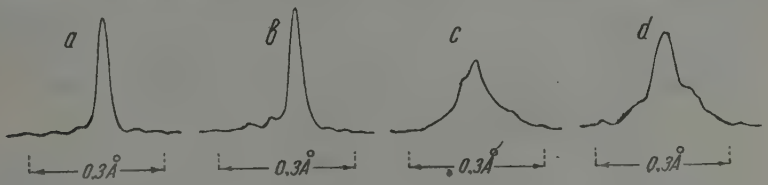


Fig. 1.

The spectra of scattering of *p*-cresol and phenol were investigated at two temperatures, viz., at room temperature (19° C), and in heating the *p*-cresol to a temperature of 140° C, and the phenol to a temperature of 174° C. The liquids were heated so as to reduce their viscosity. The viscosity of *p*-cresol is $\eta=0.2$ at 19° C, and $\eta=0.006$ at 140° C. The viscosity of phenol is $\eta=0.12$ at 19° C, and $\eta=0.006$ at 174° C. It was thus possible, by heating the liquids, to reduce the viscosity of *p*-cresol 30 times and that of phenol 20 times.

Fig. 1 shows the microphotometric records of the spectrograms obtained by us. The results were as follows. Owing to the high viscosity of the cold liquids, their spectrograms show only a very strong depolarized undisplaced line. Fig. 1,*b* shows a microphotometric record of the spectrum of cold *p*-cresol (the spectrum of cold phenol differing in no essential particulars from the latter has not been reproduced here).

It may easily be seen that this spectrum is essentially identical with that of the exciting light ($\lambda=4358$ Å) of a mercury lamp (Fig. 1, *a*).

Heating of the liquids, as well as the consequent reduction of viscosity, brings out substantial changes in the spectrum. The undisplaced line decreases in intensity and a strong depolarized continuous background appears in its neighbourhood. This is visible both in the spectrogram of heated cresol (Fig. 1, *c*) and in that of heated phenol (Fig. 1, *d*).

These experiments are thus seen to confirm the above considerations as to the connexion between the continuous background of the Rayleigh line and the depolarized portion of the undisplaced component of the triplet, as well as the dependence of both on the viscosity of the liquid.

Physical Institute.
University of Leningrad.

Received
6. II. 1941.

REFERENCES

[1] E. Gross, Nature, **126**, 201, 400, 603 (1930); **129**, 722 (1932). [2] Л. Мандельштам, Журн. Русск. физ.-хим. об-ва, **58**, 831 (1926). [3] L. Brillouin, Ann. de Phys., **17**, 88 (1922). [4] P. Debye, ibid., **39**, 789 (1912). [5] L. Landau und G. Placzek, Phys. ZS. d. Sowjetunion, **5**, 172 (1934). [6] K. Birus, Phys. ZS., **37**, 548 (1936). [7] C. V. Raman and B. V. Raghavendra Rao, Nature, **139**, 584 (1937); **141**, 242 (1938); C. V. Raman and C. S. Venkateswaran, Nature, **142**, 791 (1938). [8] E. Gross, C. R. Acad. Sci. URSS, XXVIII, 786 (1940).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

PHYSIK

ÜBER DIE DREHUNG DER POLARISATIONSEBENE IM GEKRÜMMTEN LICHTSTRAHL

Von W. W. WLADIMIRSKI

(Vorgelegt von S. I. Wawilow, Mitglied der Akademie, den 24. I. 1941)

Wie S. M. Rytov(1) gezeigt hat, erfolgt bei der Ausbreitung des Lichtes in einem Medium mit langsam von Punkt zu Punkt variierendem Brechungsindex eine Drehung der Polarisationsebene in bezug auf das natürliche Tripel, das von den Einheitsvektoren der Tangente $\mathbf{t}$, der Normale $\mathbf{n}$ und der Binormale $\mathbf{b}$ des gekrümmten Strahls gebildet wird. In der vorliegenden Notiz wird die geometrische Seite dieser Erscheinung eingehender analysiert.

In der Näherung der geometrischen Optik gelten die Vektoren des elektrischen und des magnetischen Feldes als orthogonal. Die durch beide Vektoren gelegte Ebene ändert ihre Orientierung im Raum, wobei sie jedoch stets senkrecht auf dem Strahl steht. Es sei $\mathbf{e}$ der Einheitsvektor in der Richtung des elektrischen Feldes und $\mathbf{h}$ der Einheitsvektor in der Richtung des magnetischen Feldes:

$$\mathbf{e} = \mathbf{n}\cos\varphi + \mathbf{b}\sin\varphi; \qquad \mathbf{h} = -\mathbf{n}\sin\varphi + \mathbf{b}\cos\varphi. \tag{1}$$

Wie Rytov gezeigt hat, rotiert das Tripel $\mathbf{t}$, $\mathbf{e}$, $\mathbf{h}$ in bezug auf das natürliche bewegliche Tripel des Strahls $\mathbf{t}$, $\mathbf{n}$, $\mathbf{b}$, wobei der Differentialquotient des Drehungswinkels gleich ist der Torsion der Kurve

$$\frac{d\varphi}{ds} = \frac{1}{\tau}. \tag{2}$$

Hier bedeutet φ den Winkel zwischen dem elektrischen Vektor und der Normale des Strahls, s — den nach dem Strahl abgelesenen Bogen, τ — den Torsionsradius des Strahls. Einer derartigen Rotation entspricht der Girationsvektor $\mathbf{t}/\tau$, der längs der an den Strahl gelegten Tangente gerichtet ist, da der Differentialquotient eines beliebigen, mit $\mathbf{t}$, $\mathbf{e}$, $\mathbf{h}$ starr verbundenen Vektors $\mathbf{a}$ der Bogenlänge nach gleich $\left[\frac{\mathbf{t}}{\tau}\mathbf{a}\right]$ ist, wenn man als Bezugssystem das bewegliche Tripel $\mathbf{t}$, $\mathbf{n}$, $\mathbf{b}$ benutzt. Die Änderung des Vektors $\mathbf{a}$ in bezug auf das unbewegliche Bezugssystem setzt sich zusammen aus seinem Differenzialquotienten in bezug auf das bewegliche System $\mathbf{t}$, $\mathbf{n}$, $\mathbf{b}$ und der Rotation des beweglichen Tripels $\mathbf{t}$, $\mathbf{n}$, $\mathbf{b}$ relativ zum unbeweglichen Bezugssystem:

$$\frac{d\mathbf{a}}{ds} = \left[\frac{\mathbf{t}}{\tau}\mathbf{a}\right] + [\omega\mathbf{a}] = [\mathbf{g}\mathbf{a}] \tag{3}$$

wo ω der Darboux'sche Girationsvektor ist, der die momentane Rotation des natürlichen Tripels **t**, **n**, **b** bestimmt; es ist

$$\omega = \frac{\mathbf{b}}{\rho} - \frac{\mathbf{t}}{\tau} \tag{4}$$

wo ρ den Krümmungsradius des Strahls bedeutet. Somit heben sich die Tangentialkomponenten der Girationsvektoren gegenseitig auf, und der resultierende Girationsvektor des Tripels **t**, **e**, **h** relativ zum unbeweglichen Bezugssystem

$$\mathbf{g} = \frac{\mathbf{b}}{\rho} \tag{5}$$

ist immer längs der Binormale des Strahls gerichtet und von der Torsion unabhängig. Die von Rytov ermittelten Differentialquotienten der Vektoren **e** und **h** im beweglichen Bezugssystem

$$\frac{d\mathbf{e}}{ds} = [\mathbf{ge}] = -\frac{\mathbf{t}}{\rho}\cos\varphi; \quad \frac{d\mathbf{h}}{ds} = [\mathbf{gh}] = \frac{\mathbf{t}}{\rho}\sin\varphi \tag{6}$$

sind längs der Tangente an den Strahl gerichtet.

Obwohl die momentane Winkelgeschwindigkeit des Tripels **t**, **e**, **h** immer senkrecht auf dem Strahl steht, und folglich keinerlei Rotation der Feldvektoren um den Strahl herum stattfindet, so wird im allgemeinen Falle die Polarisationsebene doch nicht jedesmal in ihre ursprüngliche Lage zurückkehren, sobald die Tangente an den Strahl mit ihrer ursprünglichen Richtung zusammenfällt, da ja die Umdrehungsachse **b** immerzu ihre Orientierung im Raum ändert, falls der Strahl Torsionsbewegungen ausführt. Den vollen Drehungswinkel der Polarisationsebene kann man leicht mittels einer geometrischen Hilfskonstruktion ermitteln. Zu diesem Zwecke tragen wir die den verschiedenen Punkten des Strahls zugehörenden Tangentialvektoren aus einem gemeinsamen Anfang auf. Nach Massgabe des Fortschreitens unseres Punktes längs des Strahls beschreibt dann das Ende des Tangentialsvektors eine gewisse Kurve Γ auf der Einheitssphäre. Die Vektoren **e** und **h** können, da sie senkrecht auf **t** stehen, in der an die betreffende Sphäre gelegten Tangentialebene aufgetragen werden. Die Änderungen von **e** und **h** längs des Strahls entsprechen der pseudoparallelen Verschiebung dieser Vektoren auf der Sphäre längs der Kurve Γ, da ihre Differentialquotienten längs **t**, d. h. längs der Normale der Sphäre gerichtet sind (es handelt sich hier um Differentialquotienten im dreidimensionalen Raum). Besitzt die Tangente an den Strahl im Anfangs- und im Endpunkt die gleiche Richtung, so ist die Kurve Γ in sich geschlossen. Bei der pseudoparallelen Verschiebung längs der geschlossenen Kurve erfahren die Vektoren bekanntlich eine Drehung um den Winkel Θ, der gleich ist dem Flächenintegral nach der Gauss'schen Krümmung über das von der Kurve Γ begrenzte Gebiet. Im Falle der Einheitssphäre ist dies Integral gleich dem körperlichen Winkel Ω, den der vom Vektor **t** gebildete Kegel einschliesst; das Vorzeichen des Winkels Θ wird bestimmt durch die Richtung der Umgehung des körperlichen Winkels

$$\Theta = \Omega. \tag{7}$$

Wenn also die Tangente an den Strahl zu ihrer ursprünglichen Richtung zurückkehrt, dreht sich die Polarisationsebene um einen Winkel, der dem von der Tangente beschriebenen körperlichen Winkel gleicht.

Stellt der Strahl eine ebene Kurve dar, so ist $\Theta = 0$, und bei der Rückkehr des Strahls in die Ausgangsrichtung nehmen die Feldvektoren ihre frühere Lage ein. Der vom elektrischen Felde und der Tangentialfläche gebildete Winkel bleibt in diesem Falle längs des ganzen Strahls unverändert, da hier die Kurve Γ mit der geodätischen Linie, d. h. mit dem Bogen des grossen

in der Strahlenebene gelegenen Kreises zusammenfällt, und der Winkel zwischen dem Vektor und der Tangente an die geodätische Linie bei der pseudoparallelen Verschiebung keine Änderung erfährt [dies geht auch unmittelbar aus (2) hervor, da für eine ebene Kurve $\frac{1}{\tau}=0$].

Besitzen die Tangenten an den Strahl im Anfangs- und im Endpunkt verschiedene Richtungen, so liegen die Vektoren **e** und **h** dieser Punkte in verschiedenen Ebenen, und die Polarisationszustände können nicht unmittelbar verglichen werden. Zweckentsprechend müsste man beim Vergleich der Polarisationszustände den Strahl in seine frühere Richtung zurückführen, entweder mittels einer ebenen Kurve oder durch Reflexion an einem idealen Spiegel, der die Polarisation nicht beeinflusst. Die Kurve Γ wird dabei durch den Bogen des grossen Kreises geschlossen, und der Drehungswinkel der Polarisationsebene kann wieder nach Formel (7) ermittelt werden. Diese Definition verliert ihre Eindeutigkeit nur in dem Falle, wenn die Tangenten im Anfangs- und im Endpunkt antiparallel sind: ein beliebiger grosser Kreis, dessen Ebene durch **t** geht, kann dann dazu dienen, die Kurve Γ zu schliessen.

Die Einführung eines lokalen Masses für die Drehung der Polarisationsebene im gekrümmten Strahl stösst auf bestimmte Schwierigkeiten. Wir bringen weiter unten drei mögliche Definitionen, von denen eine jede gewisse Mängel aufweist.

1. Man setzt als Girationskonstante den Differentialquotienten des vom elektrischen Vektor und der Normale gebildeten Winkels $\frac{d\varphi}{ds}$, d. h. die Winkelgeschwindigkeit der Feldvektoren bei ihrer Drehung in bezug auf das natürliche Tripel, dividiert durch die Lichtgeschwindigkeit im betreffenden Punkt. Gemäss (2) ist im isotropen Medium mit langsam variierendem Brechungsindex die Drehung der Polarisationsebene gleich der Torsion des Strahls. Gerade dieser Definition der Girationskonstante entspricht die Formel Rytovs für die Differenz der Fortpflanzungsgeschwindigkeiten des rechts- und linkspolarisierten Lichtes bei zirkularer Polarisation. Der volle Drehungswinkel der Polarisationsebene ist jedoch nicht gleich dem Integral der Girationskonstante nach dem Strahl, da zu dieser Grösse noch der Drehungswinkel des natürlichen Tripels relativ zum unbeweglichen Bezugssystem hinzugefügt werden muss. In dem praktisch besonders wichtigen Falle des geradlinigen Strahls werden die Richtungen der Normale und der Binormale und zugleich damit auch die Torsion der Kurve unbestimmt. Somit fehlt ein unmittelbarer Übergang von diesem Girationsmass zu der für den geradelinigen Strahl üblicherweise eingeführten Grösse.

2. Etwas bequemer ist es, die Tangentialkomponente des Girationsvektors des Tripels **t**, **e**, **h** als Mass der Drehung der Polarisationsebene anzunehmen. Bei dieser Definition ist die Drehung in einem optisch inaktiven Medium mit langsam variierendem Brechungsindex gleich Null. Der volle Drehungswinkel braucht trotzdem nicht Null zu sein und wird durch (7) bestimmt. Für den geradelinigen Strahl fällt diese Definition mit der allgemein üblichen zusammen.

3. Wird man sich dahin einigen, die Messung der Polarisation stets bei einer und derselben Richtung der Tangente $\mathbf{t}_0$ vorzunehmen, so lässt sich für jeden Punkt des Strahls das Azimut des elektrischen Vektors Φ definieren als der Winkel zwischen einer festen, auf $\mathbf{t}_0$ senkrecht stehenden Richtung und dem elektrischen Vektor, den man auf der Sphäre längs des Bogens des grossen Kreises in den Punkt $\mathbf{t}_0$ verschiebt. Der Differentialquotient eines solchen Azimuts nach der längs des Strahls gemessenen Bogenlänge ist in unserem Falle gleich dem Verhältnis zwischen dem Flächeninhalt eines unend-

lich kleinen Dreiecks mit den Ecken $\mathbf{t}_0, \mathbf{t}, \mathbf{t}+d\mathbf{t}$ auf der Einheitssphäre (Abb. 1) und dem entsprechenden Bogendifferential ds. Dieser Differentialquotient kann als Mass der Drehung der Polarisationsebene angesetzt werden, da das Integral dieser Grösse gleich ist dem vollen Drehungswinkel $\Phi_2 - \Phi_1 = \Theta$, insofern $\mathbf{t}$ in die Ausgangslage zurückkehrt. Diese Grösse stellt jedoch kein lokales Mass der Drehung der Polarisationsebene im wahren Sinne des Wortes dar, da sie wesentlich von der Hilfsrichtung $\mathbf{t}_0$ abhängt. Für das integrale Mass der Drehung tritt diese Schwierigkeit nicht auf, da die volle Änderung des Azimuts längs der geschlossenen Kurve Γ von $\mathbf{t}_0$ nicht abhängt. Am zweckmässigsten ist wohl die zweite Definition.

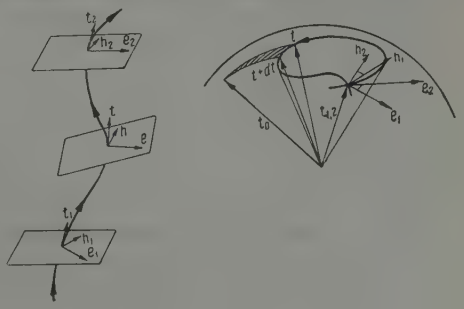


Abb. 1. Der gekrümmte Strahl und die Spur des Tangentialvektors auf der Einheitssphäre.

Die hier analysierte Erscheinung ist experimentell nicht beobachtet worden. Jedoch darf wohl mit der Möglichkeit des Auftretens derartiger Effekte bei der Untersuchung inhomogener Medien in gekreuzten Nicolschen Prismen gerechnet werden: aus der Aufhellung des Feldes darf man nicht eindeutig auf die Anisotropie des Mediums schliessen, da die Änderung der Polarisation durch die Krümmung des Strahls bewirkt sein kann.

Universität Moskau.

Eingegangen
den 7. II. 1941.

ZITIERTE LITERATUR

[1] S. M. Rytov, C. R. Acad. Sci. URSS, XVIII, 263 (1938); Тр. Физ. ин-та АН СССР, 2 (1), 109 (1940).

Übersetzt von D. Penner.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

PHYSIK

EIN ANALOGON DES PHOTOGRAPHISCHEN SCHWARZSCHILD-EFFEKTES IN REINEM CHLORSILBERKRISTALL

Von P. MEIKLJAR

(Vorgelegt von A. N. Terenin, Mitglied der Akademie, den 25. I. 1941)

Gegenwärtig darf es auf Grund zahlreicher Forschungen als festgestellt gelten, dass als Träger der Eigenschaften des latenten photographischen Bildes kolloide Ag-Teilchen auftreten, die sich bei Lichteinwirkung in den Emulsionskörnern ausscheiden. Die vorliegende Arbeit verfolgt den Zweck, die Grösse der bei Lichteinwirkung entstehenden Kolloide, sowie den Einfluss einiger Faktoren auf deren Wachstum zu untersuchen.

§ 1. Zur Grössenbestimmung der Teilchen dient bei uns die Untersuchung der optischen Eigenschaften von kolloiden Lösungen und zwar ihrer Absorption (1); wir vergleichen die auf spektral-photometrischem Wege ermittelten Messergebnisse mit den Werten, die sich aus den Formeln der Optik der kolloiden Lösungen (Rayleigh-Mie) errechnen lassen. Als Faktoren, die das Wachstum der Teilchen beeinflussen, wählten wir die Beleuchtungsstärke auf dem Kristall und die Belichtungsdauer. Als Versuchsobjekte dienten AgCl-Schichten, die zunächst zwischen zwei Glasplatten gegossen und alsdann freigelegt wurden. Zur Belichtung der AgCl-Schichten diente ein 25-A Lichtbogen, der in verschiedenen Abständen vom Kristall angeordnet wurde und somit verschiedene Beleuchtungsstärken auf dem Kristall erhalten liess.

Die Beleuchtungsstärke wurde mittels einer photoelektrischen Zelle kontrolliert. Da bei geringen Abständen zwischen Lichtbogen und Kristall störende Wärmewirkungen zu erwarten sind, so wurde die Temperatur des Kristalls mittels eines Thermoelements (Kupfer-Konstantan) gemessen; stieg die Temperatur um 2—3° an, so wurde die Belichtung eingestellt. Die Lichtabsorption wurde durch Messungen vor und nach der Belichtung nach dem Verfahren der objektiven spektral Photometrie (Monochromator von Schmidt und Hänsch, Selen-Photozellen) ermittelt.

§ 2. Abb. 1 zeigt die Absorptionskurven des Systems Ag-AgCl, die nach den Formeln von Rayleigh und Mie berechnet wurden*. Bei der

* Aus der Optik der Kolloidmetalle ist bekannt, dass die Lage der Absorptionskurven durch den spektralen Gang der optischen Konstanten (n und $n\varkappa$) des Kolloidstoffes und des Lösungsmittels bedingt wird. In den experimentellen Angaben über die optischen Konstanten der Metalle sind grosse Divergenzen vorhanden (1). So liegen für Silber zwei Varianten optischer Konstanten vor (nach Hagen und Rubens einerseits und nach Minor und anderen Verfassern andererseits). Unseren Berechnungen sind die Angaben von Minor zu Grunde gelegt. In der Arbeit von Savostianova und Toporec (2) wurden die Angaben von Hagen und Rubens für das System Ag-AgBr verwertet. Das ganze Kurvensystem ist bei dieser Variante im Vergleich zu unseren Kurven nach der Seite der kürzeren Wellen

Vergrösserung der Teilchendurchmesser verschieben sich die Kurven nach der Seite der grösseren Wellenlängen und nehmen dabei flachere Form an. Bei noch grösseren Teilchen mit einem Durchmesser von 100 mμ ist eine nahezu gleichmässige Absorption in einem breiten Wellenbereich zu erwarten. Die betreffenden Versuchsergebnisse werden in Abb. 2 wiedergegeben. Wir haben drei Serien von Absorptionskurven der AgCl-Kristalle in Abhängigkeit von der Belichtungsdauer aufgenommen, wobei innerhalb jeder Serie die Beleuchtung konstant gehalten wurde. Es wurde eine Reihe aufeinanderfolgender Expositionen vorgenommen; nach jeder Exposition erfolgte eine Messung der Absorption. Abb. 2 zeigt die zweite Serie der Absorptionskurven. Wie man hier sieht, ver-

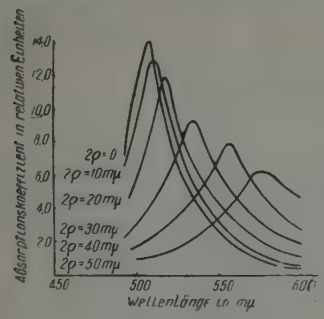


Abb. 1.

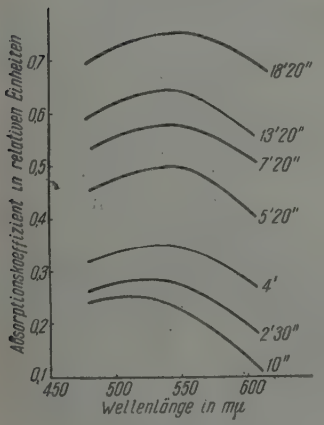


Abb. 2.

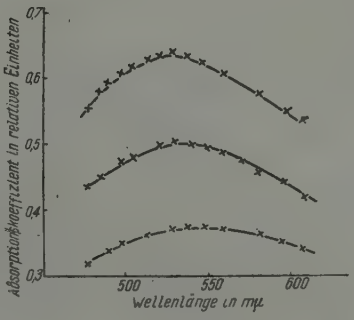


Abb. 3.

schiebt sich das Maximum der Absorptionskurve bei wachsender Belichtungsdauer, aber konstant gehaltener Beleuchtungtstärke nach der Seite der längeren Wellen. Dies Resultat besagt, dass die Vergrösserung der gesamten Belichtungsdauer zum allmählichen Anwachsen der Kolloidteilchen führt.

Abb. 3 zeigt vergleichsweise je eine Kurve aus jeder Serie für eine und dieselbe Belichtungsdauer im Betrage von 4 Minuten. Hier sieht man, dass mit der Steigerung der Beleuchtungsstärke auf dem Kristall sich als Ergebnis photochemischer Prozesse kleinere Teilchen bilden.

§ 3. Durch Vergleich der Kurven in Abb. 2 und 3 mit den Kurven in Abb. 1 kann man (unter Berücksichtigung des in der Fussnote gemachten Vorbehalts) die Grösse der Kolloidteilchen abschätzen. Abb. 4 zeigt die Abhängigkeit des auf diese Weise ermittelten Teilchendurchmessers von der gesamten Belichtungsdauer für alle drei Serien (Tabelle).

---

verschoben, und zwar beträgt diese Verschiebung bei der Kurve für das feinste Kolloid 30 mμ. Die Unsicherheit in den Zahlenwerten der optischen Konstanten stellt die Bestimmung der Absolutwerte der Teilchendurchmesser beim Vergleichen der berechneten bzw. der experimentellen Angaben in Frage; auf die Einschatzung ihrer relativen Grössen wirkt sich dieser Umstand jedoch nicht aus.

Durch Differentiation der Kurven von Abb. 4 lässt sich die Abhängigkeit der Wachstumsgeschwindigkeit der Teilchen von der gesamten Belichtungsdauer ermitteln. Das Ergebnis ist in Abb. 5 dargestellt.

Serie	E in Lux	2ρ in mμ		
		$t =$ = 1 Min.	$t =$ = 2 Min.	$t =$ = 4 Min.
I	15800	18	23	27
II	5760	25	27	30
III	1440	29	30	32

§ 4. Aus dem Obigen geht somit hervor, dass: 1) die im AgCl-Kristall bei Lichteinwirkung ausgeschiedenen Silberteilchen grösser werden a) bei Abnahme der Beleuchtungsstärke und b) bei Vergrösserung der gesamten Belichtungsdauer; sowie dass 2) bei steigender Belichtungsdauer die Teilchen nach einem Gesetze wachsen, das dem Exponentialgesetz nahe kommt*.

Somit zeigt sich hier die Ungleichwertigkeit der Faktoren der Zeit und Beleuchtungsstärke bei der Ausscheidung der Silberteilchen im Kristall eines Halogensalzes durch Lichteinwirkung. Diese Erscheinung kann man als ein Analogon des bekannten photographischen Schwarzschild-Effektes betrachten, wonach die Schwärzungsdichten in der entwickelten Schicht zweier Aufnahmen verschieden sind, wenn sie bei einer und derselben Lichtmenge, aber verschiedener Beleuchtungsstärke gemacht worden sind.

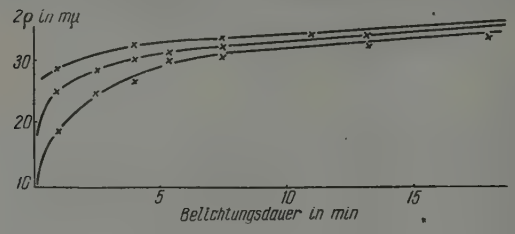


Abb. 4.

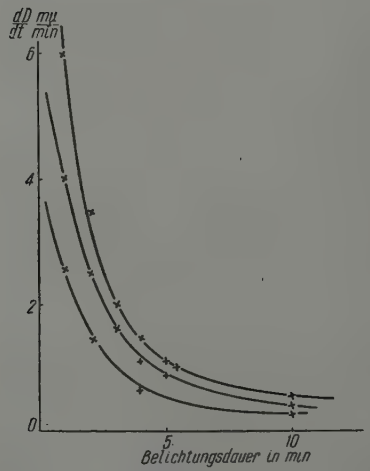


Abb. 5.

Rabinowitsch und Bagdassarjan (4) haben gezeigt, dass diese Erscheinung mit der Veränderung der Silbermenge zusammenhängt, die sich photochemisch aus dem Bromsilber ausscheidet, wenn man bei verschiedener Beleuchtungsstärke eine und dieselbe Lichtmenge auf die photographische Emulsion auffallen lässt. Unsere Ergebnisse weisen auf einen weiteren Faktor hin, der sich auf die Menge des Silbers in der entwickelten photographischen Schicht auswirken kann, nämlich den verschiedenen Dispersionsgrad der photochemisch ausgeschiedenen Teilchen. Es scheint eine gewisse optimale Teilchengrösse des latenten Bildes zu existieren, die bei der gegebenen Quantität der aufgefallenen Lichtenergie die maximale Dichte der entwickelten Schicht zustandekommen lässt. Dies wird durch die Untersuchung von Arens (5) bestätigt, der zufolge gegebene Silbermenge in der Emulsionsschicht im Falle eines grösseren Dispersionsgrades eine grössere Dichte nach der Entwicklung ergibt.

* Dies Ergebnis fällt mit den Angaben Coopers für Chromalaun zusammen [vergl. die Übersicht von Garner (3), S. 1183].

Bei der Durchführung der Analogie zwischen diesen Effekten im reinen Kristall des Halogen-Silbersalzes einerseits und in der photographischen Emulsion andererseits, ist zu berücksichtigen, dass bei der Belichtung in den vorliegenden Fällen gewöhnlich wesentliche Unterschiede vorliegen. So wurden in unseren Versuchen sowohl grosse Beleuchtungsstärken, als auch grosse Expositionen verwendet, um solcherart möglichst kleine Teilchen in messbarer Konzentration zu erhalten. Die Beleuchtungsstärke betrug tausende Lux, die Expositionsdauer besass die Grössenordnung einiger Minuten, während es bei der Exponierung der Photoschichten genügt mit etwa 30 Lux innerhalb $^1/_{50}$ Sek. zu arbeiten, um nach der Entwicklung die Dichte =1 zu erzielen.

Derselbe Einwand bezieht sich auch auf die Auswertung der bekannten Versuche von Hilsch und Pohl ([6]). Aus ihren Angaben geht tatsächlich hervor, dass zwar die Intensität des Lichtstromes in ihren Versuchen von derselben Grössenordnung war, wie bei der Herstellung photographischer Bilder (z. B. bei AgBr 225, resp. 100 erg/sec. cm^2), dass dafür aber die Expositionszeiten wesentlich divergierten: 15 Sek. im ersten Falle und ca $^1/_{50}$ Sek. im zweiten. Auf Grund unserer Angaben können wir den Schluss ziehen, dass die Teilchengrösse im ersten Falle weitaus grösser sein muss (das Maximum der Absorptionskurve des Kristalls liegt bei den Versuchen von Hilsch und Pohl bei 560 mμ, was unter Berücksichtigung des obigen Vorbehalts einem Teilchendurchmesser $2\rho=40$ mμ entspricht).

Zusammenfassend kann folgendes gesagt werden: wollten wir tatsächlich im Gebiete des latenten Bildes arbeiten, so wäre es notwendig, sich genau an die Belichtungsbedingungen in diesem Gebiet zu halten, d. h. zugleich geringe Beleuchtungsstärke und geringe Expositionsdauer zu verwenden.

§ 5. Die Bildung feinerer Teilchen bei der Belichtung des lichtempfindlichen Korns mit intensiverem Lichtstrom ist vom Standpunkt der zurzeit populären Gurney-Mott'schen Theorie ([7]) des latenten Bildes unverständlich. Nimmt man an, dass die Silberteilchen sich auf schon vorhandenen Silberkeimen (oder, bei Emulsionen, auf Keimen von Ag_2S) bilden, so müsste im Falle schwacher Beleuchtung (die einer geringen Elektronenkonzentration im Leitungsband des Kristalls entspricht) eine beträchtlichere Grösse der Teilchen dafür sprechen, dass nicht alle Silber-, bzw. Ag_2S-Teilchen bei diesen Bedingungen Elektronen binden, was jedoch unverständlich erscheint. Nimmt man dagegen an, dass im Verlauf der Entstehung des latenten Bildes das Elektron sich nicht «weit» von den Stellen seiner Auslösung (den «atomaren Zentren») entfernt, bzw. also nicht in das Leitungsband gelangt, so dürften die in der vorliegenden Arbeit aufgezeigten Gesetzmässigkeiten verständlich werden, da bei grösserer Beleuchtungsstärke eine grosse Konzentration atomarer Zentren zustandekommt, deren Koagulation zur Bildung feinerer Teilchen führen muss.

Zum Schlusse spreche ich Dr. Kravez und Dr. Savostianova meinen Dank für ihre wertvollen Hinweise aus.

Eingegangen
den 28. I. 1941.

ZITIERTE LITERATUR

[1] М. В. Савостьянова, Усп. физ. наук, **22**, 1, 168 (1939). [2] M. Savostianova and A. Toporec, C. R. Acad. Sci. URSS, II, 225 (1934). [3] Гарнер, Усп. химии, **7**, 1753 (1938). [4] Rabinowitsch und Bagdassarjan, ZS. f. Phot., **32**, 110 (1933). [5] Arens, Veröffentl. AGFA, III, 32 (1933). [6] Hilsch und Pohl, ZS. f. Physik, **64**, 606 (1930). [7] Gurney and Mott, Proc. Roy. Soc., A, **164**, 151 (1938).

Übersetzt von D. Penner.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

PHYSICS

THE ELECTRO-OPTICAL EFFECT OF KERR IN SOLUTIONS OF *p*-AZOXYANISOL

By N. TOLSTOY and V. TSVETKOV

(Communicated by A. N. Terenin, Member of the Academy, 24. I. 1941)

To understand the structure of anisotropic liquids one has to know the structure of their molecules; in particular it is important to know the angle between the dipole and the longitudinal axis of the molecule.

Various authors have established the following:

1. Molecules of *p*-azoxyanisol in an anisotropic liquid phase in an electric field set their long axes perpendicular to the field (provided there arises no microturbulent motion) ([1],[2]).

2. The anisotropic liquid does not contain great molecular groups (swarms) having each a dipole moment as a whole. If such groups do exist, the dipole moments of the molecules which compose the group compensate each other ([2],[3]).

3. The dielectric constant of *p*-azoxyanisol is greater when parallel to molecular axes, than when perpendicular to them, $\varepsilon_{\parallel} < \varepsilon_{\perp}$ ([4]).

4. The vector of dielectric polarisation of *p*-azoxyanisol (contrary to, say, dibenzalbenzidin) has no phase shift with the vector of the electric field, if the latter rotates at a rate of $\sim$1 rot./sec. ([5]).

5. The molecule of *p*-azoxyanisol has a considerable dipole moment (2.3×10^{-18} CGSE) ([6],[7]), and the orientation contributes to the dielectric polarisation more than the polarisation.

These dielectric properties of *p*-azoxyanisol and its behaviour in the electric field may, apparently, be explained by assuming an anisotropy of freedom of rotation; the molecule turns round the long axes easier than round the short ones. The molecules of a swarm turn round their long axes under the influence of the electric field upon the dipoles of the molecules, trying to orient themselves with their dipoles along the field ([5],[8],[9]).

But if so, the dipole axis should be perpendicular to the long molecular axis, or, at least, the angle between these axes should be considerable. The value of this angle can be determined by means of the constant of Kerr's electro-optical effect (if the optical and electrical ellipsoids of the molecule are known). Zadoc-Kahn ([10]) tried to measure Kerr's constant in melted *p*-azoxyanisol, but succeeded only in determining the sign (a positive one) of the constant, from which she erroneously inferred that the dipole is perpendicular to the molecule (in this case the constant would be negative).

We have measured the Kerr-effect of solutions of *p*-azoxyanisol in: 1) benzene, 2) tetrachlormethane. The birefringence was measured with the compensator of Brace ([11]). An overglowed cinema lamp of 50 watt served as a light source. Measurements were carried out in white and yellow light.

The converging beam was focussed on the half-shadow plate. The path difference in the compensator (its constant) was $\sim\lambda/_{100}$.

The effect in solutions was compared with the effect in solvents. Kerr's constant referred to one molecule of *p*-azoxyanisol K_{azox} was calculated in asssuming that Kerr's effect of diluted solutions of polar substance in a non-polar is additive:

$$K_{azox} = \frac{K_{solution}(1+\eta) - K_{solvent}}{\eta} \tag{1}$$

where η is the ratio of the molecular concentration of components, $K_{solvent}$ is Kerr's constant of 1 molecule of liquid solvent, $K_{solution}$ — the same referred to 1 «conventional» molecule of the solution (it is assumed that the number of molecules in 1 cm³ is the same in solutions of different concentrations; this is admissible for diluted solutions).

It was assumed, that molecules of *p*-azoxyanisol in the solution can be considered as a gas, i. e. formula (2) was used

$$K_{azox} = B_{azox}N = 3\pi N(\theta_1 + \theta_2) \tag{2}$$

where N is the number of molecules in 1 cm³ and

$$\left.\begin{aligned} \theta_1 &= \frac{1}{45kT}[(a_1-a_2)(b_1-b_2)+(a_2-a_3)(b_2-b_3)+(a_3-a_1)(b_3-b_1)] \\ \theta_2 &= \frac{1}{45k^2T^2}[(\mu_1^2-\mu_2^2)(b_1-b_2)+(\mu_2^2-\mu_3^2)(b_2-b_3)+(\mu_3^2-\mu_1^2)(b_3-b_1)] \end{aligned}\right\} \tag{3}$$

$\mu_{1,2,3}$ are components of the dipole moment, $a_{1,2,3}$ are components of the electrical polarisibility, $b_{1,2,3}$ are components of the optical polarisibility.

The molecular concentration of the solution was determined from the specific weight by means of a pycnometer (in assuming that the volumes of components of the solution are additive) and calculated from the formula

$$\eta = \frac{\mu d_1 (D-d)}{\mu_1 d (d_1 - D)} \tag{4}$$

where μ, μ_1 are molecular weights, d, d_1 — specific weights of the components, D—the specific weight of the solution. The volume of the pycnometer is ~ 5 cm³. The results are given in Tables 1 and 2.

Table 1

Solution of *p*-azoxyanisol in Benzene

$\eta \times 10^4$	71	98	99	110	116	120	123	141	145	188
$K_{C_6H_6} \times 10^{33}$	7.81	8.86	8.79	7.58	7.74	7.48	8.20	8.34	9.06	8.70

Table 2

Solution of *p*-azoxyanisol in CCl_4

$\eta \times 10^4$	33	44	51	55	59	84	84	87	92	106
$K_{CCl_4} \times 10^{33}$	4.0	3.4	1.9	2.7	2.5	1.9	1.4	5.5	2.3	4.8

The mean values: $K_{C_6H_6} = (0.83 \pm 0.05) \times 10^{-32}$
$K_{CCl_4} = (0.31 \pm 0.11) \times 10^{-32}$

$K_{C_6H_6}$ and K_{CCl_4} are Kerr's constants for 1 molecule of *p*-azoxyanisol obtained from solutions of *p*-azoxyanisol in C_6H_6 and CCl_4, respectively.

Assuming an axial symmetry of the molecule, we can obtain from (2) and (3) the angle between the dipole and the axis of the molecule

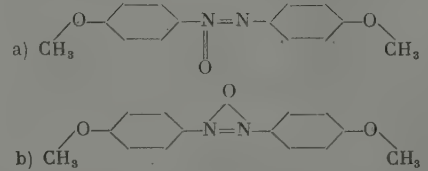
$$\alpha = \operatorname{arcot} \sqrt{\frac{K - K_{\min}}{K_{\max} - K}}. \qquad (5)$$

K is the measured constant of Kerr; $K_{\min}$ is Kerr's constant which *p*-azoxyanisol would have, if its dipole were perpendicular to the axis of molecule; $K_{\max}$ — the same in case of parallel situation of the dipole and the axis of molecule.

As it follows from (5), α depends upon K in such a way that if α is of order of $\pi/4$, the error in determining K contributes little to the error in α. On the contrary, if α is near to 0 or 90°, the error in K gives a great error in α.

To calculate $K_{\min}$ and $K_{\max}$ from (2) and (3) we used, beside the value of the dipole moment ($\mu = 2.3 \times 10^{-18}$), the data of Chatelain ([12]) who measured the birefringence of a solid crystal of *p*-azoxyanisol. According to his data, $b_1 - b_2 = 1.80 \times 10^{-23}$, $b_3 \approx b_3$. The difference of the electric polarisibilities $a_1 - a_2$ was approximatively laid equal to $b_1 - b_2$ (i. e., the difference of the optical polarisibilities).

Thus, the values obtained for K are: $K_{\min} = -0.89 \times 10^{-32}$; $K_{\max} = -2.79 \times 10^{-32}$. Using these values and the data of Tables 1 and 2, we calculated by means of (5) $\alpha_{C_6H_6} = 47°00' \pm 40'$; $\alpha_{CCl_4} = 55°10' \pm 1°50'$. It is known that the chemical formula of *p*-azoxyanisol can be written in two different ways, either a) or b), according to the two possible structures of the azoxy-group:



In the case of a) the dipole has with the axis an angle where $0° < \alpha < 90°$; in the case of b) it is perpendicular to the axis. The results given above show that a) takes place in reality.

Briegleb ([13]) and Stuart ([14]) have shown that (1) is not justified in liquids. This is clearly illustrated in our case by the difference of data, obtained for different solvents. Therefore the calculation of a molecule is irreproachable only when based on measurements in vapour.

However, measurements in vapour of *p*-azoxyanisol present extreme, if not unsurmountable difficulties, owing to the very small density of vapour within the temperature range where the substance does not decompose. Therefore, even rather rough measurements in solution, giving, nevertheless some idea of the approximate attitude of the dipole in the molecule, are of use.

Physical Institute.
State University. Leningrad.

Received
28. I. 1941.

REFERENCES

[1] Friedel, Ann. de Phys., **19**, 273 (1922). [2] Fréedericksz und Zwetkoff, Acta Phys.-Chimica URSS, **3**, 879 (1935). [3] Zwetkoff, ibid., **6**, 865 (1937). [4] Kast, An. der. Phys., **73**, 145 (1924). [5] Zwetkoff, Bull. Acad. Sci. URSS (1941). [6] Ebert und Gartel, Phys. ZS., **28**, 786 (1927). [7] Errera, ibid., **29**, 426 (1928). [8] Kast und Meier, Physica, **4**, 957 (1937). [9] Цветков, Уч. зап. пед. ин-та им. Герцена, **10**, 33 (1938). [10] Zadoc-Kahn, C. R., **200**, 822 (1935). [11] Szivessy, Handbuch der Phys., XIX, 945, 960, 965 (1928). [12] Chatelain, Bull. Soc. Franc. Minér., **60**, 280 (1937). [13] Briegleb, ZS. f. phys. Chem., Abt. B, **14**, 97 (1931); **16**, 249 (1932). [14] Stuart, ZS. f. Physik, **63**, 533 (1930).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

PHYSIQUE

SUR LA MESURE DU COEFFICIENT DE TEMPÉRATURE DE LA TENSION SUPERFICIELLE DU MERCURE

Par A. M. DIDENKO et N. L. POKROVSKY

(Présenté par S. I. Wawilow, de l'Académie, le 13. II. 1941)

Un grand nombre d'études consacrées à la mesure de la tension superficielle du mercure ne permettent pas de définir, sans ambiguïté, sa grandeur, qui varie, d'après les données publiées, de 340 dynes/cm à 520 dynes/cm. Vu que les mesures de la tension superficielle font partie des expériences

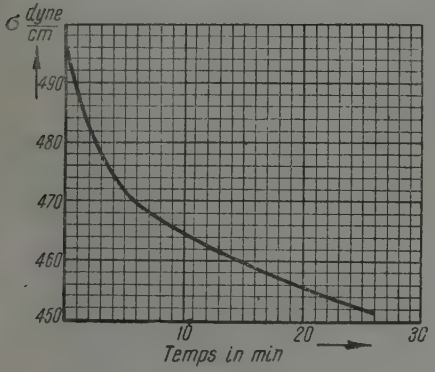


Fig. 1.

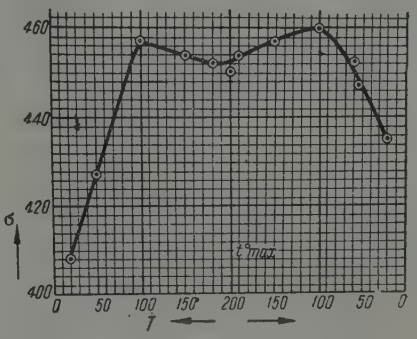


Fig. 2.

très délicates, on peut considérer comme moins sûres les données plus anciennes. Mais d'autre part, les plus récentes études, quoique bien effectuées, ont produit des résultats contradictoires.

Les mesures de la tension superficielle du mercure, effectuées dans l'air (nous n'examinons pas ici les résultats obtenus dans l'atmosphère de différents gaz, car cela est une question spéciale qui est assez vaste par elle même) donnent des résultats suffisamment concordants. Un rôle important dans ces mesures joue le temps de formation de la surface du mercure.

Si le temps est suffisamment petit et atteint l'ordre de quelques secondes, la valeur de la tension trouvée sera maximum. Ainsi, L. Convers (1) a trouvé pour la période de 10 sec. la grandeur 494 dynes/cm et pour la valeur limite de 28 min.—450 dynes/cm. Fig. 1 montre l'influence du temps sur la valeur trouvée dans l'air. La seule explication possible, ommise par plusieurs auteurs,

consiste dans les phénomènes d'adsorption, ayant lieu sur la surface du mercure.

Dans le vide, on devrait s'attendre à des valeurs de σ plus élevées, plus uniformes et faciles à regagner dans les conditions données. Mais en réalité nous voyons le contraire: les valeurs de σ dans le vide sont en majeur partie considérablement moindres, et de plus, se contredisent beaucoup. Dans la table sont réunis les résultats obtenus par différents auteurs.

Auteur	Date	T. S. dans levide	Méthode employée
I			
Cenas	1913	460	Poids des gouttes
Hagemann . .	1914	472	Vibration du jet
Harkins-Ewing	1920	476	Poids des gouttes
Hogness . . .	1921	465	Pression dans les gouttes
Eredale . . .	1924	475	Poids des gouttes
Birkumshaw .	1928	480	Pression dans les gouttes
Cook	1929	515	Large goutte
Burdon	1932	488	» »
Brown	1933	473	Poids et forme des gouttes
Bradley . . .	1934	500	Forme des gouttes
II			
Stökle	1898	435	Rayon de courbure
Kahläne . . .	1902	431	Ondes capillaires
Schmidt . . .	1911	435	Vibration du jet
Palacios . . .	1920	402	Pression dans les gouttes
Popesko . . .	1921	436	Forme des gouttes
Richards-Boyer	1921	432	» » »
Iredale	1924	410	Poids des gouttes
Oppenheimer .	1928	437	Dépressions capillaires
Kernaghan . .	1932	438	Forme des gouttes
Sauer.-Schmidt	1933	420	» » »
Sementchenko .	1936	410	Pression dans les gouttes

La première partie de la table contient les valeurs élevées (460–515 dynes/cm); la seconde partie—les petites valeurs (410—440 dynes/cm). Il est tout naturel de supposer, comme nous l'avons fait dans le cas des mesures effectuées dans l'air, que la surface du mercure, nouvellement formée dans le vide, soit un bon adsorbeur pour toutes sortes de gaz, qui puissent se trouver à la surface du mercure, même dans le cas du vide assez parfait. Les sources de ces gaz peuvent être différentes, par exemple, les parois de verre (les gaz retenus par occlusion, les vapeurs d'eau) de même que les vapeurs des lubrifiants utilisés pour les robinets. Mais d'autre part, l'étude de l'influence produite par ces substances (vapeur d'eau, différents gaz) a montré qu'elle est insignifiante et peut être négligée [Kassel (2) a étudié l'influence de H_2O, Sauerwald (3)—celle des lubrifiants].

Contrairement aux mesures dans l'air, dans le vide le temps de formation d'une goutte n'influe pas sur la tension superficielle. Il en résulte, que si même l'adsorption a lieu, elle passe momentanément. L'adsorption a certainement lieu en cas d'utilisation de certaines méthodes, car il suffit que la température atteigne 100—150°, pour que σ augmente visiblement. L. Convers (4) a trouvé, sans chauffage, la valeur 417 dynes/cm et avec chauffage 442 dynes/cm. Le résultat analogue a été trouvé par nous. Les mesures que nous avons effectuées par la méthode décrite dans nos premiers communiqués (5), ont donné sans chauffage une valeur très petite, à savoir 410 dynes/cm. Par suite, les mesures ont été effectuées avec chauffage et ont donné les résultats présentés sur la fig. 2. La courbe obtenue peut être expliquée par la désorption, favorisée par le chauffage, ce qui cause la croissance de σ.

Il est intéressant d'observer, que la valeur primitive ne se reconstituait que le lendemain.

En tenant compte des résultats obtenus, nous avons proposé la construction de l'appareil et la méthode de mesure, libre en grande partie des défauts communs aux autres méthodes de mesure. Cet appareil, présenté sur la fig. 3, est formé de deux tubes A et B réunis comme par un lien avec C. Dans la partie inférieure l'on a un flotteur m qui est bien ajusté à la partie conique

S du tube A. Le flotteur m se termine, en bas, par un tube mince contenant un morceau de fer doux. La partie supérieure du flotteur est munie d'un capillaire. Lorsque l'appareil est rempli de mercure, le flotteur occupe la position indiquée sur la fig. 3. Le mercure arrive par le tube A jusqu'au capillaire, le passe très lentement et se dirige vers le tube B en passant dans C, qui se termine dans le tube B par un capillaire vertical K. Le tube A a dans sa partie supérieure un autre flotteur, également en fer doux, entouré du verre et flottant librement dans le mercure.

σ a été mesuré par la méthode de Kantor. La pression nécessaire était créée par l'immersion, dans le mercure, du flotteur supérieur à l'aide d'un électro-aimant N_2. L'augmentation de la pression était très lente grâce au flotteur m ayant un capillaire. On la mesure avec un cathétomètre par un tube manométrique h. Au moment de l'apparition de la première goutte, le flotteur m était attiré vers le bas par l'aimant N_1, et l'on coupait le courant dans le circuit de l'aimant N_2, de façon que le second flotteur se trouvait libre. Les tubes A et B se communiquaient, de cette façon, par un large passage, la pression se nivelait momentanément et la formation des gouttes se terminait. La méthode de production de la pression par le moyen décrit a été empruntée chez B. P. Bering.

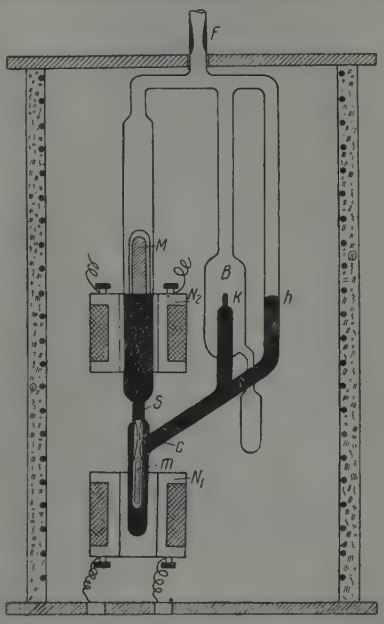


Fig. 3.

Avant le commencement de l'expérience, l'appareil était chauffé pendant 8—10 heures à la température de 350—400° C. Ensuite, on y introduisait une quantité nécessaire de mercure, après quoi l'appareil était de nouveau chauffé déjà avec le mercure pendant les mêmes heures et ensuite en état chaud fut déssoudé.

Les données obtenues par nous sont présentées sur la fig. 4. Ils résultent de nombreuses mesures qui ont été analysées par la méthode statistique*. La dépendance de la température s'exprime: $\sigma = 459.10 - 0.20\ t$. La droite de la fig. 4 est tracée d'après cette équation. Toutes les données expérimentales, indiquées par les cercles, se grouppent bien autour de la droite tracée avec l'écart, ne dépassant jamais un pour cent.

On voit également sur la fig. 4 les résultats obtenus par Hogness ([6]), considérés par nous comme les plus sûrs de toutes les données expérimentales. Ces résultats peuvent être décrits par l'équation:

$$\sigma = 467 - 0.043\,(t+39) - 0.000383\,(t+39)^2.$$

L'écart de la dépendance linéaire peut s'expliquer par la désorption croissante, avec la température, du gaz des parois de l'appareil. Ce gaz est adsorbé par les gouttes formées de mercure et abaisse la tension superficielle.

L'avantage de la construction décrite est ce que nous avons pu effectuer avec la même quantité de mercure de nombreuses mesures aux températures

* Les calculs de généralisation statistique ont été effectués par A. M. Tourbine.

différentes, en embrassant tout le domaine étudié. De plus, on peut affirmer que la formation d'une goutte de mercure (après un chauffage durable et bien réglé de l'appareil, d'une part, et de l'appareil avec le mercure ensemble, d'autre part) avait lieu sur la surface de séparation des phases de mercure liquide et de vapeur de mercure, ce qui est très important, si l'on tient compte de ce qui a été dit ci-dessus.

Il nous semble que l'influence des gaz étrangers était pratiquement exclue, ce qui est confirmé, d'une part, par les valeurs élevées et stables de σ égales

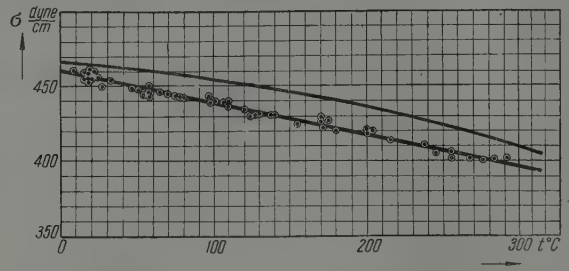


Fig. 4.

pour 20° à 455 dynes/cm, et d'autre part, par le caractère régulier de la dépendance de température (fig. 4).

Nous exprimons, avec plaisir, notre reconnaissance au chef du Laboratoire Scientifique de l'Institut Pédagogique V. K. Sementchenko, qui a été l'initiateur de l'étude exposée, et à N. D. Rozhkova pour sa collaboration dans la partie expérimentale.

Institut Pédagogique de Moscou.
Laboratoire de Physique Théorique.

Manuscrit reçu
le 14. II. 1941.

LITTÉRATURE CITÉE

[1] L. Convers, J. Chim. Physique, **36**, 175 (1939). [2] H. M. Kassel, ZS. f. Elektrochem., **37**, 642 (1931). [3] Sauerwald-Schmidt, ZS. anorg. Ch., **213**, 310 (1933). [4] L. Convers, Diss., Nancy (1938). [5] Б. П. Беринг и Н. Л. Покровский, Журн. физ. химии, VII, № 4 (1936). [6] Hogness, J. of Am. Ch. Soc., **42**, 1621 (1921).

Traduit par S. Tager.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

TECHNICAL PHYSICS

SOUND ABSORPTION BY MULTILAYER RESONANCE SYSTEMS

By V. S. NESTEROV

(Communicated by S. I. Wawilow, Member of the Academy, 29. XII. 1940)

1. The production of systems with high sound absorption coefficients is of great technical importance; particularly, such systems are needed to weaken the echo from the dome of the Great Hall of the Palace of Soviets. S. N. Rschevkin proposed as one of the possible solutions for this problem, the use of resonance sound absorbing systems ([1–4]). The simplest kind of resonance absorber is one or several thin sheets with a large number of perforations set up some distance apart from one another in front of the reflecting wall. The disadvantage of resonance absorbers with small numbers of layers (1 — 2) is their narrow range of absorption. Thus, a one-layer system ([4]) can absorb 90% and more of normally incident sound within a range of one octave, a two-layer system ([5])—within a range of 2—2.5 octaves, and a three-layer one ([6])—within a range of 3—4 octaves. In papers cited below ([4,5,6]) it has been shown that effective absorption within a frequency range of 100—4000 cycles/sec. (5.5—6 octaves) can be accomplished only by increasing the number of layers to four and more.

2. Let us consider a system of n perforated sheets, in front of a reflecting wall at a distance of L_i from one another (Fig. 1). Suppose that the orifices are round, have an area of σ_i and are arranged in the form of a regular lattice so, that each one of them takes up Σ cm² of the whole area of the absorber. The conductivity of each perforation is K_i and its mass is

$$m_i = \frac{\sigma_i^2 \rho}{K_i}. \qquad (1)$$

The orifices are damped with a thin layer of a porous substance (wire netting, light fabric, punctured tinfoil, etc.) the friction of which is r_i Ω/cm² *.

If the wave-length $\lambda \gg \sqrt{\Sigma}$, then m_i and r_i can be averaged and considered uniformly distributed all over the surface of the i-th layer. The average values of these parameters in fractions of ρc per surface area of 1 cm² are $\frac{\Sigma}{cK_i}$ and $\frac{r_i \Sigma}{\rho c \sigma_i}$. Let

$$M_i = \frac{\Sigma}{K_i} \qquad (2) \qquad \text{and} \quad R_i = \frac{r_i \Sigma}{\rho c \sigma_i}. \qquad (3)$$

* S. N. Rschevkin and S. T. Ter-Osipiantz ([7]) showed that within a wide frequency range the specific friction of the wire nettings might be considered constant; the absolute values of friction per cm² can be measured after a tube-method worked out by S. T. Ter-Osipiantz.

The values M_i and R_i together with the depths L_i are parameters of the absorber.

It can be shown that for a plane wave with normal incidence the full impedance at the entrance to the system can be expressed as a continued fraction:

$$Z = Z_1 + \cfrac{1}{\sin^2(kL_1)\left\{Z_2 + \cfrac{1}{\sin^2(kL_2)\left[Z_3 + \cfrac{\cdots\cdots}{\cdots\cdots + \cfrac{}{\sin^2(kL_{n-1})\,Z_n}}\right]}\right\}} \tag{4}$$

where k is the wave number, and the values of Z_i equal:

$$\left.\begin{aligned} Z_1 &= R_1 + j(kM_1 - \operatorname{ctg} kL_1) \\ Z_2 &= R_2 + j(kM_2 - \operatorname{ctg} kL_2 - \operatorname{ctg} kL_1) \\ &\cdots\cdots\cdots\cdots\cdots \\ Z_i &= R_i + j(kM_i - \operatorname{ctg} kL_i - \operatorname{ctg} kL_{i-1}) \end{aligned}\right\} \tag{5}$$

Formula (4) holds true until the values of friction and mass on the surfaces of the plates can no longer be averaged (appr. up to $f \approx 3-4000$ cycles/sec. when $\sum = 10$ cm^2, or up to $f \approx 8-10000$ cycles/sec. when $\sum = 1$ cm^2). Those frequencies whose arguments kL_i are multiples of $n\pi$ are exceptions to this statement.

More elaborate analysis shows that at these frequencies the i-th air layer does not take part in the wave process within the absorber. Therefore it is quite desirable that none of the dimensions L_i be equal to or multiple of one another.

Formula (4) is not limited to the system in Fig. 1. It holds good as well for constructions consisting of: a) perforated sheets with various $\sum_i$ and with the perforations not opposite one another; b) sheets with an irregular lattice of perforations (the lattice may e. g. be in the form of an ornament); c) layers of fabric (constructions used for the first time in the Trokadero theatre in 1909, and later studied by Wintergerst and especially perfected by Bedell); d) layers of metallic screen (G. D. Malyuzhinetz' construction); e) thin layers of a porous material l_i in thickness with air spaces L_i wide, where $l_i \ll L_i$.

Systems (a) and (d) are calculated by means of formula (4) exactly as described below. For systems (b), (c) and (e) the values M_i and R_i must be determined preliminarily by measuring the material in a one-layer system. Then the values of M_i and R_i, found experimentally, are substituted in formulae (4) and (5), and the nomogram in Fig. 2 (see below) is used.

On the basis of K. A. Vital's papers ([8], [9]) formula (4) can be generalized so as to cover the case of oblique incidence of a plane wave. This can be accomplished by merely multiplying all the parameters R_i, L_i and M_i in (5), as well as in (4) by the cosine of the angle of incidence. In this case the values R_i and M_i become, generally speaking, functions of the angle of incidence, depending upon the structure of the layers. The general appearance of the formula for input impedance remains identical with (4) in the case of oblique incidence.

Having calculated the impedance of the system from (4) we determine the coefficient of absorption α by means of well known formula.

3. In calculating multilayer systems it is convenient to use the graphical method. Substituting the arguments kL_i in (5) for ρ_i, we can write:

$$\left.\begin{aligned} Z_1 &= R_1 + j\left(\frac{\Sigma}{k_1 L_1}\varphi_1 - \operatorname{ctg}\varphi_1\right) \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ Z_i &= R_i + j\left(\frac{\Sigma}{k_i L_i}\varphi_i - \operatorname{ctg}\varphi_i - \operatorname{ctg}\varphi_{i-1}\right) \end{aligned}\right\} \quad (6)$$

The partial reactances, according to (6), are determined with a pair of compasses and a nomogram, on which the curve ctg φ and the pencil of straight lines $\frac{\Sigma}{KL}\varphi$ are plotted.

The inversion $\frac{1}{\sin^2\varphi_{n-1}(Z_i+\ldots)}$ can be accomplished by making use of the nomogram in Fig. 2, which consists of the curve $\sin^2\varphi_{n-1}$, inversion circles and two pencils of straight lines. Having determined the impedance Z_n from (6) for the frequency under question, we mark off its components in the upper half of the nomogram and φ_{n-1} in the lower scale. From the point corresponding to the abscissa of φ_{n-1} we go upwards until we meet the curve $\sin^2\varphi$, and then move along the horizontal to $O'R_n$.

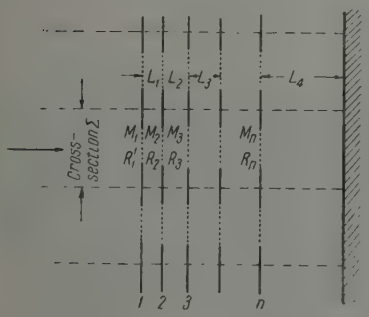


Fig. 1.

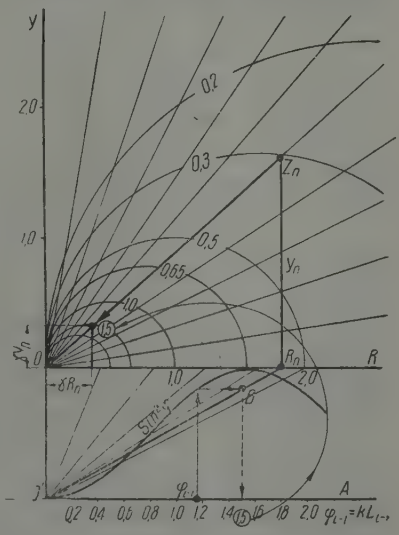


Fig. 2.

The intersection point B gives $R_n\sin^2\varphi_{n-1}$ whose value can be marked off on axis OR or $O'A$. Then we look for the inversion circle whose number equals the abscissa of point B. The intersection point of this circle with OZ_n gives the components of the unknown value $\frac{1}{\sin^2\varphi_{n-1}Z_n}$. Suppose they are γR_n and γY_n. Then we find $R_{n-1}+\gamma R_n$ and $Y_{n-1}-\gamma Y_n$ (the components of $Z_{n-1}+\frac{1}{\sin^2\varphi_{n-1}Z_n}$), measure them off from the upper part of the nomogram, mark φ_{n-2} and repeat the same operation until we get to the first layer. The last operation gives the full input impedance of the system. The absorbing coefficient can be calculated conveniently with the help of a nomogram of absorption circles ([4]), a family of which are drawn on a complex plane. The absorption of the system for a given frequency can be found easily if Z is marked off on the same plane, from the parameter of the corresponding circle.

As an example, Fig. 3 gives results of a calculation of a four-layer resonance system, in which it was attempted, while building it, to attain $\alpha \geqslant 0.9$ in a range of 100 — 4000 cycles/sec. As the figure shows, this attempt was successful. The parameters of the system are: $L_1 = 11$ cm; $L_2 = 15$ cm; $L_3 = 19$ cm; $L_4 = 32$ cm (full depth 77 cm); $M_1 = 0.33$ cm; $M_2 = 1.05$ cm; $M_3 = 2.4$ cm; $M_4 = 8$ cm; $R_1 = 0.35$; $R_2 = 0.5$; $R_3 = 0.6$; $R_4 = 1.0$. The parameters were selected roughly at first, and differed from those given. They were selected on the basis of considerations as to the required depth of the absorbing layer ([6]), the non occurrence of the length L_i,

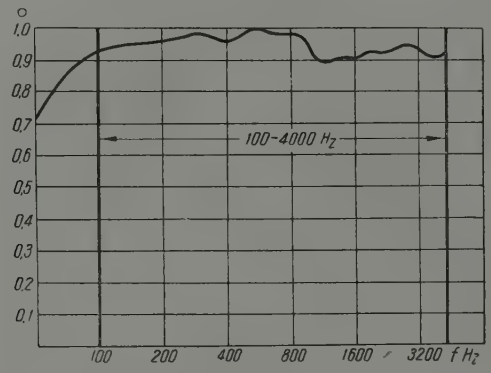


Fig. 3.

multiple to one another, the gradual decrease in partial frequencies towards the wall ([6]), and the damping of the partial systems required ([5]). Calculation of Z and α from rough parameters made it possible to make corrections and adapt the absorption to meet the requirement. We have grounds to believe that the system mentioned can be improved by changing the parameters a little more, and then its absorption will finally approximate 95% in a range of 100 — 4000 cycles/sec. The parameters M_i and R_i can easily be accomplished in various ways. The diameters of the orifices and the perforation coefficients of various sheets can be found by merely selecting the specific friction and thickness of the sheets.

The author expresses his gratitude to Prof. S. N. Rschevkin for a number of valuable suggestions.

P. N. Lebedev Institute of Physics.
Laboratory of Acoustics.
Academy of Sciences of the USSR. Moscow.

Received
28. XII. 1940.

REFERENCES

[1] С. Н. Ржевкин, Тр. Акуст. Ком. АН СССР, вып. 3 (1940). [2] С. Н. Ржевкин, ЖТФ, VI, 2103 (1936). [3] S. N. Rschevkin, C. R. Acad. Sci. URSS, XVIII, 25 (1938). [4] S. N. Rschevkin, ibid., XXII, 9 (1939). [5] В. С. Нестеров, ЖТФ, IX, 19 (1939). [6] В. С. Нестеров, там же, X, 8 (1940). [7] S. N. Rschevkin and S. T. Ter-Osipiantz, Journ. of Phys. (1941). [8] К. А. Виталь, ЖТФ, X, 12 (1940). [9] К. А. Виталь, Тех. отч. отд. АК СДС (1939).

Translated by D. Soboloff.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

GEOLOGY

GRAVITATIONAL ANOMALIES AND PROCESSES OF TECTOGENESIS

By V. V. BELOUSSOV

(Communicated by P. I. Stepanov, Member of the Academy, 24. I. 1941)

A. D. Arkhangelsky ([1,2,3]) and others have published a critical survey of fundamental literature on the application of gravimetrical data to the solution of important problems of geology, and a number of highly interesting and valuable papers on the correlation of gravity anomalies and internal structure of the earth crust on the territory of the USSR.

A new comparison of modern data on the distribution of gravity anomalies on the surface of the earth corroborates the opinion of A. D. Arkhangelsky as to the existence of two causes of such anomalies. The structrue of outer portions of the earth crust, which is responsible for the distribution of light and heavy rocks near the surface, is one of them. Fluctuations of the force of gravity in connexion with this factor are usually insignificant and of local character. The depth of occurrence of the crystalline basement and thickness of friable deposits are in the case under consideration the main factors directly affecting the gravitational regime.

If, however, instead of local disturbances the general scheme of distribution of gravitational conditions on the surface of the planet is considered, there would be found out a number of circumstances which deprive the mentioned factor of its dominant position. Thus e. g. premountainous hollows [marginal subgeosynclines ([4])] are characterized by negative anomalies*. This holds true both in the case of the earth crust subsidence, being compensated by an accumulation of friable deposits (continental foredeeps), and in the case of absence of such deposits (deep oceanic hollows encircling the island arcs). On the other hand, young ovals of subsidence show positive anomalies despite the fact that they sometimes contain thick beds of accumulated friable material (the Mexican Gulf). Such comparisons indicate that the basic large scale plan of the distribution of gravitational anomalies on the planet is due to other factors which are functionally associated with the geotectonic regime of the given area.

It should be supposed, in accordance with A. D. Arkhangelsky's views ([1]), that the geotectonic factor of gravitational anomalies is brought about, principally, by the phenomena of contraction and expansion in the internal material of the earth.

When assuming that the earth crust follows in its vertical movements the contraction and expansion of the subcrustal material, it is natural to

* Anomalies, as here used, mean total or isostatic anomalies.

expect that the force of gravitation diminishes in the points of uplift and increases in those of subsidence. Fenoscandia and Northern Germany, respectively, bear witness to the presence of such relations. Other gravitational data display, however, more complicated relations.

Negative anomalies in the zones of active subsidence of the earth crust (marginal subgeosynclines, Meinesz belts) may be explained on the assumption that in the case of contraction of the internal parts of the earth, the crust sags, following this contraction, but lags somewhat behind, thus inevitably causing a drop in density directly under the crust; this deficiency of density is revealed on the surface in the form of a gravitational deficit.

This hypothesis presumes a considerable strength of the earth crust, which is responsible for the latter's reaction to internal contraction with a certain lag in time.

Lagging of the movement of the earth crust behind the contraction is revealed, however, only in the case of quick and energetic internal contractions. In the case of a slow contraction the earth follows it without any lagging, and only insignificant positive anomalies are produced (Northern Germany).

This lagging of the earth crust behind internal contractions and the resulting formation of density deficiency zones directly under the crust should be considered responsible for negative anomalies also in large grabens of Eastern Africa and the upper Rhein.

What happens when the subsidence finally comes up with the volume of contraction of internal masses? An answer to this question is furnished by the Red Sea which, although belonging to the system of East-African grabens, possesses positive anomalies. Here the subsidence occurred on a large scale, totally compensated the internal contraction, and the zone of density deficiency ceased to exist. Positive anomalies are representative of the excessive density of the subcrustal material and also of the surface effusions of volcanic rocks.

Under conditions, similar to those of the Red Sea, there are found young ovals of subsidence (the Mediterranean Sea, the Mexican Gulf, the Caribbean Sea, etc.) formed as a result of a recent collapse in the place of a preceding intense upheaval, which completed the Alpian folding. The upheaval was, perhaps, associated with an expansion of abyssal masses which was consequently replaced by contraction, the result being the collapse of ovals. Positive anomalies are indicative of the internal contraction being compensated by the earth crust subsidence.

The collapse after a considerable upheaval, completing the folding, affects mostly the internal portions of folded zones (4). From the above said it is evident that such a collapse should be preceded by the appearance of strong negative anomalies. Since the epoch of Alpian macro-oscillations is still going on, an attempt may be made to find on the globe such areas which are «mature» for collapse, that has not yet occurred. Such a state is typical of the Pamir-Alay region and the central part of the folded region of the North American Cordilleras, which are characterized by strong negative anomalies. On the geological time scale these regions are «on the eve» of their collapse and conversion into ovals of subsidence.

Positive anomalies characteristic of Alpian folded zones (beyond the limits of internal negative regions) are explained by an inspissation of the earth crust as a result of folding and metamorphism and also «weighting» of the crust with intrusions. But in older folded zones the magnitude of positive anomalies diminishes, and they are replaced by negative anomalies. This evolution of the gravitational regime demonstrated by a comparison of anomalies in Alpian, Hercynian and Caledonian folded zones, is caused, on the one hand, by a gradual yielding of the gravitational effect of the surface inspissation to the effect of a continuing growth of internal density defi-

ciency (the latter may be based on an observed phenomenon of progressive uplifting of ancient folded regions, occurring, in all probability, under the conditions of expansion of internal masses). On the other hand, the rôle of surface inspissation diminishes as a result of erosion which gradually carries away the inspissated layer.

In conclusion, the following scheme of development of the gravitational regime of the geosynclinal zone during the geotectonic cycle may be established.

The geosynclinal subsidence is accompanied with strong negative anomalies which are caused by lagging of the earth crust movements behind the contraction of subcrustal masses and also by the resulting appearance of a zone of «sucking in» deinspissation directly under the crust.

After inversion, the central geosynclines, where the folding accompanied with intrusions is formed, should tend to positive total anomalies (but to Bouge's negative anomalies), since here the inspissation takes place near the surface and expansion occurs in the depth. Gravitational forces grow together with a further development of central geosynclines and intensification of folding and intrusive activity. Toward the end of the main epoch of folding formation the negative (total) anomalies are retained only in marginal subgeosynclines, where still proceeds an intense subsidence of the earth crust.

Later on, in the internal parts of the folded zone there appear strong negative total anomalies. This occurs under the conditions of intense contraction of internal masses which replaces the preceding expansion. Owing to this contraction, the earth crust, as a result of its strength increased by folding and intrusions, experiences a noticeable lagging behind the internal masses, this leading to the appearance of a subcrustal zone of density deficiency. At a certain moment the crust ceases to support its own weight, collapses along fault fissures (plastic downwarping is hindered in this case by an increased strength of the crust due to folding and intrusions) and forms subsidence ovals. Since collapsing compensates for the preceding subcrust deinspissation of the material, these ovals show positive anomalies.

The peripheral portions of folded zones do not participate, as a rule, in collapsing during the epoch of macro-oscillations and continue a slow uplifting movement during later geological periods. The initially existing positive total anomalies gradually diminish on account of a continuing internal expansion, on the one hand, and erosion carrying away the surface inspissated layer, on the other, and may be replaced by negative anomalies.

Radium Institute.
Academy of Sciences of the USSR.
Leningrad.

Received
13. XI. 1940.

REFERENCES

[1] А. Д. Архангельский, Тр. XVII сессии Межд. геол. конгр. 1937 г., I (1939). [2] А. Д. Архангельский, Тр. н.-и. ин-та геол. и мин., вып. 1 (1933). [3] A. D. Arkhangelsky, Bull. Acad. Sci. URSS, sér. géol., No. 4 (1937). [4] V. V. Beloussov, C. R. Acad. Sci. URSS, XXIX, № 5—6 (1940).

Translated by P. Rappoport.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

MINERALOGY

CHEMICAL COMPOSITION OF LOVOZERO EUDIALYTITES

By N. A. ELISEEV and E. A. SVERJINSKAYA

(Communicated by A. E. Fersman, Member of the Academy, 11. II. 1941)

The eudialytites of the Lovozero tundras, which consist mainly of eudialytes, are very interesting from a standpoint of natural concentrations of zirconium. Eudialytites occur mostly in the form of irregular pockets in eudialyte luyavrites and other rocks of the Lovozero alkali plutone. In appearance eudialytites are very beautiful medium-grained rocks, red, pink or brown in colour, rarely dark, almost black, consisting mainly of eudialyte. According to calculations made by N. A. Eliseev (1), the content of eudialyte in eudialytites varies from 60 to 70% (Table 1).

	1	2	3	4	5	6	7	8
Eudialyte	62.0	61.2	70.0	75.0	65.5	74.7	65.1	74.7
Nepheline	3.2	1.0	1.7	—	0.2	—	13.2	0.1
Sodalite	—	very little			—	—	4.1	—
Feldspars	8.3	1.8	5.2	—	11.5	4.3	4.3	1.0
Aegirine	13.8	17.7	11.0	12.3	3.4	11.9	8.0	7.4
Alkali amphibole	1.2	1.5	2.6	8.3	7.2	3.3	—	1.0
Ore minerals	5.6	6.8	1.7	4.4	6.9	0.6	—	2.5
Zeolites	5.0	6.3	7.4		2.0	0.1	5.3	13.3
Lamprophyllite	0.9	3.7	0.4	—	3.3	5.1	—	—
Ramsayite	—	—	—	—	—		—	—

Note. 1, 2, 3—samples from the Chivruai deposit; 4, 5, 6—samples of eudialytite from the Kuftuai deposit; 7, 8—samples from the Vavnbed deposit (the figures in the table are weight per cents).

The samples for calculation were selected with a high content of eudialyte. Eudialyte is more or less evenly concentrated in various sections of the same pocket. These eudialytite pockets have well defined boundaries. They range from a few cm to several tens of meters in cross-section.

It is curious to note that the eudialyte content varies very little in comparison with the contents of other minerals.

In order to characterize the eudialytites chemically, three samples from different deposits of the Lovozero tundras were analyzed by E. A. Sverjinskaya: 1) Eudialytite from the Kuftuai deposit; a medium-grained rock of pinkish-red colour. 2) Eudialytite from the Chivruai deposit; a beautiful dark rock almost black. 3) Eudialytite from the Vavnbed deposit, on the west slope

of mt. Vavnbed: a pinkish brown rock of a slightly trachytoid character due to the parallel orientation of crystals of aegirine and alkali amphibole.

The results of these analyses are given in Table 2.

Table 2

	1	2	3
SiO_2	49.36	49.02	50.59
TiO_2	1.81	2.08	1.36
ZrO_2	6.76	7.46	8.68
Nb_2O_5	0.58	0.39	0.76
Ta_2O_5			
TR	1.56	1.01	1.68
Al_2O_3	6.51	2.33	3.29
Fe_2O_3	4.04	5.57	6.08
FeO	4.08	4.69	1.46
MnO	1.30	1.50	1.90
CaO	7.39	7.34	5.25
Sr		1.18*	1.05**
BaO	—	0.15	0.13
MgO	0.89	1.32	1.06
K_2O	1.65	1.38	1.35
Na_2O	10.81	8.16	10.64
Cl	0.65	0.52	0.34
F	0.11	0.11	0.04
H_2O const.	2.35	5 02	3.96
H_2O hygr.	0.35	0.82	0.45
	100.20	100.05	100.07
$-O = Cl_2 + F_2$	—0.18	—0.15	—0.09
	100.02	99.90	99.98
Specific weight	2.94	2.87	2.895

One of the distinguishing features of eudialytites is the high content of ZrO_2, TR, CaO, MnO, Na_2O, Cl in their chemical composition. This is due to the presence of large quantities of eudialyte. The ZrO_2 content in eudialyte varies from 10.5 to almost 14% (2). Niobium and tantalum are constantly associated with zirconium, and therefore it is not surprising that the pentoxides of these metals are present in amounts up to 0.76%. Nb_2O_5 and Ta_2O_5 were not put to separate quantitative analyses. Niobium prevails considerably over tantalum, which is one of the distinguishing geochemical features of the Lovozero alkali plutone.

According to N. A. Eliseev (1), eudialytites are typical pneumatolytites. From this standpoint the high concentrations of calcium, strontium and manganese in eudialytites are especially interesting, since these elements occur in very small quantities in the rocks of the Lovozero plutone. W. N. Lodočnikov (3) terms such elements and oxides, which are rare in magmatic rocks and occur in large quantities in postmagmatic formations (pneumatolytites and products of hydrothermal action), polar. Thus, calcium, strontium and manganese are in this case typical polar elements. The high chlorine and fluorine content in eudialytites emphasizes once again the pneumatolytic nature of these rocks. The large quantities of water in the eudialytites are due to the presence of zeolites which formed during the hydrothermal metamorphism of feldspars and feldspathoids.

* SrO was determined in the Spectrography Laboratory of the All-Union Institute of Geology at Leningrad. The determination showed 1.18% SrO. This quantity is subtracted from the sum CaO+SrO which equals 8.52%, in order to determine the CaO content.

** Determination of SrO in the Spectrography Laboratory of the All-Union Institute of Geology showed its content to be 1.05%.

The mineral and structural features of eudialytites are described in detail in a paper by N. A. Eliseev (1).

All-Union Institute of Geology.
Leningrad.

Received
12. II. 1941.

REFERENCES

1 Н. А. Елисеев, Зап. М. О., ч. XXIX, No 4 (1940). 2 Минералы Хиб. и Ловоз. тундр, под ред. А. Е. Ферсмана (1937). 3 В. Н. Лодочников, Тр. ЦНИГРИ, вып. 38 (1936).

Translated by D. Soboloff.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

MINERALOGY

ON THE MINERALOGY OF THE IONA IRON-ORE DEPOSIT

By G. P. BARSANOV

(Communicated by A. E. Fersman, Member of the Academy, 5. III. 1941)

In 1940 the A. P. Karpinsky Geological Museum received from A. F. Sosedko specimens of magnetite and some types of ore skarns from the Iona deposit of magnetite, located in the Iona-Kovdorozero region of the Kola peninsula (Murmansk region). These specimens, together with those of L. B. Antonov (1936), served as a material for the present work. It should be noted that the mineralogy of the Iona deposit has not been studied at all. The literary data are restricted to a paper by I. G. Bakhirev (1) (a geological-petrographical outline), K. M. Koshich (2) (petrography) and P. N. Chirvinsky(3).

The material studied by me embraces the following types of skarn and ore-deposits: 1) massive granular compact magnetites with considerable phenocrysts of fresh light-green phlogopite; 2) apatite-magnetite ore of a breccia-like texture; 3) coarsely druse-like ores composed of crystalline magnetite, frequently in excellently formed large crystals cemented together with calcite, forsterite, etc.; 4) porous apatite aggregates with caught up separate fragments of magnetite; the cavities are often filled with staffelite, and in the specimens from the oxidation zone coatings of bright-green copper phosphates are observed; 5) schorlomite-calcite skarn with forsterite, sphene, calcite and more scarce apatite and magnetite.

An examination of the above specimens under the binocular-magnifying glass and in thin-sections under the microscope permitted to separate out and establish in them, in addition to common calcite, the following minerals.

M a g n e t i t e (magnesioferrite) forms compact magnetite ores, partly breccia-like, as if cemented together with granular apatite. It forms also druse-like aggregates composed of both small and large crystals and growths, embedded in a mass of calcite, forsterite, etc. The crystals are exclusively of an octahedral habit, reaching the size from 3 to 4 cm along the edge. Of the forms, the most common are *p* (111) or a combination of *p* (111) and *d* (110), the latter form being developed only as one making obtuse the octahedron edges. The faces of large crystals are dull, uneven, with an irregular sculpture of growth, the edges being somewhat rounded. The magnetite has a deep-black colour, without any signs of oxidation. Fracture uneven, slightly conchoidal. Hardness, lustre and colour of streak usual. A microscopical study in the reflected light shows that the magnetite forms an ore-aggregate consisting of separate sinuous anhedral fields among non-ore minerals. With high magnifications in some grains typical structures of

the distectic of magnetite and ilmenite are observed, the latter forming most frequently an irregular exceedingly fine «pseudo-emulsion» intergrowth. In addition, a poikilitic intergrowth of magnetite with non-ore minerals is observed, chiefly with silicates (phlogopite and forsterite), as well as apatite. Etching reveals an allotriomorphic granular structure. Spectral analysis has revealed a permanent considerable admixture of magnesium to the magnetite, which undoubtedly indicates the presence of a certain amount of isomorphous molecules of magnesioferrite. An analogous result is given by an analysis of «an as pure as possible magnetite», cited by Bakhirev ([1]). When recalculating to magnetite, which I made according to the Fe_2O_3 content, an excess of Fe_2O_3 over FeO equal to 14.74% is obtained, which should evidently be connected with MgO. When recalculating this, a magnetite content of 52.63%, and a magnesioferrite content of 17.14% was obtained, which gives a ratio of molecules in an isomorphous mixture $Fe''Fe_2'''O_4 : MgFe_2'''O_4 = 3 : 1$. An analysis of the magnetite selected (a crystal)

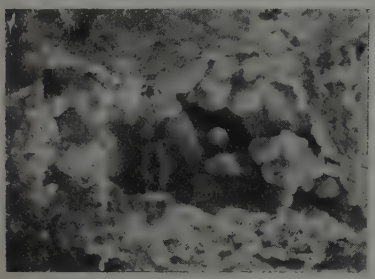


Fig. 1. Staffelite. An aggregate of stalactitic structure. Parallel and radial structure; × 20.

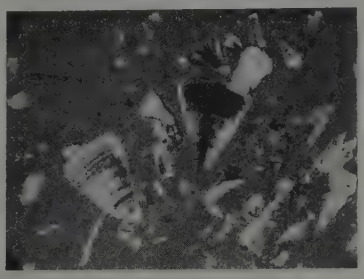


Fig. 2. Radial aggregates of staffelite (preparation in liquid 1.645); ×86.

shows a content of TiO_2, 0.82%; MgO, 6.80% (analyst N. V. Lazarev). When recalculating to ilmenite and magnesioferrite, we obtain: 1.55% ilmenite (ingrowths in magnetite); 34.0% magnesioferrite.

The above data point quite convincingly to a constant presence of isomorphous molecules of magnesioferrite and magnetite from the Iona deposit, with ratios $Fe'' Fe_2'''O_4 : MgFe_2'''O_4 = 3 : 1$ and up to 2 : 1. The latter is in agreement with the data for magnesioferrites proper, and thus the magnetites of the Iona deposit is to be partly considered as magnesioferrites, which should be, however, verified on a large quantity of material, taken from different parts of the deposit.

Forsterite is a widespread mineral in skarn formations of the Iona deposit. It forms irregular granular segregations among areas of druse-like magnetite or among fine-grained apatite-calcite aggregates in ore-skarns. The forsterite of the Iona deposit has a light-yellow or brownish colour with a characteristic greasy lustre, resembling apatite, is usually broken by a network of cracks and has a poorly developed cleavage along (010). Individual segregations of forsterite sometimes reach 2.5 to 3 cm across, but usually considerably smaller. In the thin-sections there are observed rounded and elongated prysmatic crystalline grains, poikilitic intergrowth aggregates and grains of apatite, phlogopite, calcite and magnetite. The mineral of the earliest generation is probably earlier than magnetite. The optical properties are as follows: extinction straight, biaxial, $2V \pm 90°$, sign of zone positive; $Ng \| Z$; $ng = 1.683 \pm 0.002$; $np = 1.651 \pm 0.004$; $ng - np = 0.032$. Spec-

tral analysis has revealed admixtures of Fe and weak lines of Co and Cu (see table).

Table of Spectral Analyses of the Minerals from the Iona Deposit (made at the Spectral Analysis Laboratory of the Institute of Geological Sciences of the Academy of Sciences)

Name of mineral	Intense lines. Very intense lines*	Medium lines	Weak lines	Very weak lines	Traces
Magnetite crystals	Fe*, Ti, Mg	Al	Mn, Si	Zn, V, Ca	Sn, Cu, Co
Magnetite from apatite-magnetite ore	Fe*, Mg*, Ti, Al	Mn, V	Si, Zn	Sn, Ca	Cu, Co, Mo
Forsterite	Mg*, Si	Fe	Co, Ca, Al, Mn	—	Cu
Titanclinogumite	Mg*, Si, Ti	Fe	Cu, Ca, Al, Mn	Co	—
Schorlomite	Fe*, Si, Ti, Ca	Al, Mn, Mg, Na	Zr, V	Ga, Sn	Cu, Co, Sr
Phlogopite	K*, Al*, Mg*, Si*, Na, Ba, Fe	Ca, Ti	Ga, V, Mn	Co, Sr	Cu
Staffelite	Ca, P	—	Mg, Fe, Si, Cu, Nb	Sr, Al, Mn	Be

Note. 1) Intense and very intense lines—the elements determine the composition of the mineral. Medium lines—elements giving the main admixtures. Weak, very weak lines and traces—elements which are insignificant admixtures.

2) In addition to the elements listed in the table, the minerals were analyzed for Be, As, Te, Sb, Pb, Ag, Cd, Ni, Ta, Bi, In, Ge, W, Cr, Tl, Li, which are absent in all of them.

Titanclinogumite in the specimens studied is a rare mineral. It forms irregular small granular accumulations in cracks and voids between crystals and grains of magnetite in druse-like skarn. The colour of the mineral is bright, red-brown, lustre vitreous; fracture uneven, due to a considerable fracturing of individual grains. Cleavage very poorly pronounced. Characteristic is the presence of polysynthetic twins revealed microscopically. The optical properties are as follows: extinction oblique, angle of extinction about 10°; pleochroic from red-brown along *np* to light-yellow along *ng*; biaxial positive, $2V$ about 80°; $ng=1.686\pm0.003$; interference colours bright red-green of the second order; $ng-np$ about 0.035. Spectral analysis gives intense lines of Mg and Ti (see table).

Schorlomite forms a peculiar type of schorlomite-calcite skarn and is represented in the specimens in the form of large irregular vein-like accumulations. The mineral has a pitch-black colour with a strong lustre on the fresh fracture surfaces. Streak dark-brown, hardness 7—7.5. After ignition it becomes slightly magnetic, in thin fragments being fused to a black glass. With soda it gives only a weak reaction to Mn. Opaque, translucent only in thin fragments. In microscopic preparations transparent, of a light reddish-brown colour. Isotropic, shows no optic anomalies; *n* very high, determined with difficulty in melts as lying close to 1.9. A mineral of the early generations.

Phlogopite is a widespread mineral in the contact complex of the Iona deposit. It forms small crystalline lamellar inclusions and columnar minute crystals of a hexagonal habit to 8 mm and more along the diameter. It grows with magnetite, carbonates, forsterite, apatite, etc. In the specimens from the oxidation zone passes into hydromica, becomes opaque, of a light mother-of-pearl green or light-bronze colour, loses elasticity of the foliae and acquires the property of swelling upon heating. It is very frequent that

such hydromicas are stained by copper salts in a bright-green colour. Fresh minute crystals of phologopite are transparent, light-green, occasionally nearly colourless. Cleavage, hardness and other properties fully correspond to those of mica. Optical study shows it to be almost colourless; slightly pleochroic are only some grains, from colourless along np to weakly greenish along ng. Optically pseudo-uniaxial, sometimes with a very small $2V=2-3°$, negative; $np=1.575\pm0.002$; $ng=1.604\pm0.003$; $ng-np=0.030$. Characteristic is a screen-like direct extinction. Spectral analysis, in addition to K, Mg and Fe, has shown the presence of Ba and Ca (see table).

S p h e n e has been found only in one specimen, in the form of small separate grains and irregular growths, 2—3 mm in size. Colour dark-brown. Grains of sphene are inclosed in finely crystalline calcite-apatite-forsterite skarn, together with schorlomite. In the thin-section it shows a characteristic interference colour of the higher orders and a sharp relief. Optically positive, biaxial, $2V$ intermediate; n measured in melts = 1.89.

S t a f f e l i t e is a variety of apatite, forming nodular-stalactitic aggregates and crusts, of a composition $3Ca_3(PO_4)_2 \cdot CaF_2 \cdot CaCO_3$. P. N. Chirvinsky [3], I. G. Bakhirev [1] and others have mistaken it now for chalcedony and now for smithsonite, as which it has been described. As a rule, it occurs in specimens representing portions of primary apatite-magnetite ores exposed in the zone of hypergenesis on the southern slope of Mt. Pilkomé-Selga. The mineral is crypto- or finely crystalline, forming thin cross-fibrous veinlets or botryvidal aggregates of a light-green or brownish colour, filling voids in magnetite-apatite skarn. The size of individual nodules does not exceed 2 to 3 mm. Nodular formations have a radial structure, in the center of the aggregate the relict of a grain of crystalline apatite being usually preserved, which is gradually replaced by staffelite. The hardness is less than that of apatite (about 5), but it is possible that this is explained by a brittleness of microscopic crystals, composing the aggregates. In hydrochloric acid it slowly dissolves in the cold with a liberation of CO_2; upon heating it dissolves very quickly. It gives a very good reaction for phosphorus, as well as calcium. Under the microscope it is seen that the aggregates of staffelite are composed of thin hair-like crystals forming radial aggregates with a characteristic aggregational polarization. Separate crystals are extended along L_6 with which coincides Np, which corresponds to the optic character of apatite. The mineral is uniaxial, negative, with a straight extinction; $ng = 1.628 \pm 0.003$; $np = 1.622 \pm 0.002$; $ng - np = 0.006$. In the staffelite aggregates is observed a lit-par-lit contamination by pulverulent hydroxides of iron.

As there exists a group of similar minerals (dallite, francolite, etc.), very closely approaching one another in composition and properties, in order to assign the correct name to this mineral an analysis was made for CO_2, H_2O and F (analysts V. Danilova and P. A. Volkov): CO_2, 2.61%; F, 3.55%; $\pm$ H_2O, 1.74%; the presence of Cl has been recorded. The values obtained fully correspond to those for staffelite. The origin of staffelite should be associated in the Iona deposit with the action on apatite of carbon dioxide thermae and its gradual replacement, staffelite being always observed in paragenesis with serpentinized orthosilicates and talc, and in the center of its aggregate half-dissolved relicts of apatite grains are often encountered.

Besides the minerals described, in the Iona deposit is widespread a common greenish apatite, which seems to cement together patches of clastic magnetite. The optical and chemical analyses [1] give the usual composition of fluorapatite. Of rarer minerals the specimens contain talc forming foliated inclusions in paragenesis with magnesia silicates, and pleonaste forming intergrowths in magnetite.

The above description of some of the most interesting minerals of the Iona skarns, found by me in the specimens available, does by no means exhaust the mineralogy of this peculiar deposit. It is clear, however, that in its type the contact process of the Iona deposit approaches the magnesia-silicate type of skarns which is near to the type of processes of Fennoscandia, and has no complete analoga over the territory of this country. Of great interest is the constant geochemical presence of cobalt even in silicates, revealed by spectral analysis, which fact indicates the possibility of its concentration in some other complexes of the region, to which particular attention should be paid in the course of further study.

A. P. Karpinsky Geological Museum.
Academy of Sciences of the USSR.
Moscow.

Received
6.III.1941.

REFERENCES

[1] И. Г. Бахирев, Производ. силы Кольского п-ва, I (1940). [2] К. М. Кошич, Изв. Лгр. геол.-гидро-геодез. треста, вып. 1 (1934). [3] П. Н. Чирвинский, Производ. силы Кольского п-ва (1940).

Translated by A. Brashnina.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

PETROGRAPHY

FLUORITE IN THE LOWER PERMIAN DEPOSITS OF THE BASHKIR AUTONOMOUS SOCIALIST SOVIET REPUBLIC

By V. P. FLORENSKY

(Communicated by P. I. Stepanov, Member of the Academy, 24. I. 1941)

Fluorite was found in the Lower Permian deposits of Bashkiria while exploring the Kungurian beds of the Tuimaza oil deposit located 150 km westwards from the town of Ufa.

Over the territory of this region the Kungurian stage is represented by a complex of dolomite and anhydrite strata associated with dolomite limestones, marls (in subordinate amount), and with gypsums in the upper part of the section. There is some regularity in the alternation of these strata, which is due to a certain rhythm, peculiar to the life of the basin and largely connected with the cycles of fluvial activity, the increased supply of fresh water and, as a result, the reduced salt concentration in the basin. The beginning of each of these periods is marked by an accumulation of marls and argillaceous dolomites (the moment of a maximum freshening and an increased supply of clastic material); by the close of each such period, when the concentration of the salts had attained its maximum, precipitation of calcium sulphates took place (which among anhydrite was the prevalent and the primary one). About the middle of the period there took place the deposition of normal dolomites and peculiar dolomite-anhydritic and dolomite-gypsum rocks characterized by a simultaneous precipitation of these two compounds (probably, in accordance with the well-known reaction of Haidinger*).

Fluorite was first detected when examining the heavy fraction of the insoluble residue of Kungurian rocks. The insoluble residue was isolated in the usual way by treating the rock with a 10% hydrochloric acid solution at a low temperature. The insoluble constituents obtained after this were separated by way of decantation from the pelitic fraction, and divided with the aid of Thoulet's solution into two parts with different specific gravities (above and under 2.7). The former consisted of clastic minerals with fluorite and celestite grains, these being sometimes so abundant that they made up the largest part of the heavy fraction.

We thus succeeded in finding that fluorite was widely distributed throughout the halogen Kungurian beds of the Tuimaza, though in tenths or even hundredths of a per cent of the total mass of the rock. Later on fluorite segregations were found in common petrographic plan-parallel thin-sections as well,

* A more detailed discussion of the problems of the lithogeny of the Kungurian beds of the Tuimaza deposit is given elsewhere ([1], [2]).

though here it was rather difficult to detect owing to the small rock area covered by the thin-section. This fact points to the importance of examining the insoluble residue not only when studying clastic minerals (as has been often, though not always, correctly believed), but also to find out all the syngenetic constituents, frequently overlooked when thin-sections are made use of.

Fluorite forms grains of various sizes, averaging from 0.1—0.3 mm (Fig. 1); rectangular outlines are typical of this mineral. Sometimes the crystals show the outlines of regular hexahedron. The sequence of crystallization is determined by the fact that fluorite is usually associated with sulphates and is idiomorphous with regard to the latter. Fluorite is quite isotropic, nearly always colourless, only occasionally showing a spotty, faintly violet coloration. The determination of the refractive index made by the method of immersion in a number of grains showed the following result (mean value): $n = 1.435 \pm 0.002$.

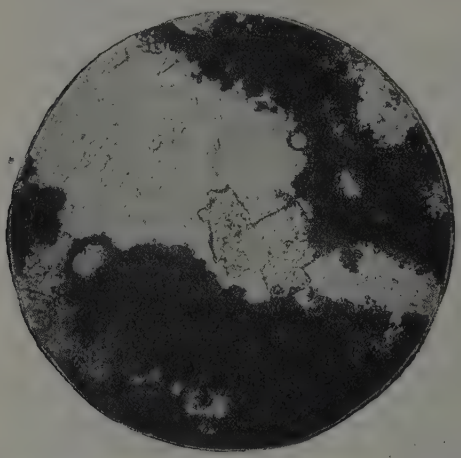


Fig. 1. Fluorite in a carbonate-sulphate rock; × 80.

Isotropy, low refractive index and numerous thin cleavage cracks intersecting the grains in different directions, are characters decisive for the diagnostics of fluorite in thin-sections. Aside from this, the presence of fluorine was confirmed by chemical analyses.

As to the genesis of fluorite, we may speak quite confidently of its pegnitogene origin and deposition as a chemical precipitate from marine waters, upon the increase of concentration of its salt content. This interpretation becomes the more probable as the whole complex of minerals of the Kungurian beds (dolomites, anhydrites, gypsum and celestite), and its attitude within the section of the Upper Paleozoic, point to a sharp increase in the salinity of the basin. For lack of space we do not dwell more on the discussion of this statement. Suffice to note that, owing to the general high concentration of salts in the basin and the impossibility there of organic life, an organic origin seems to be improbable. The same seems to be true of a fumarolic or hydrothermal origin, considering the complete absence of any alterations in the enclosing rocks and a more or less uniform distribution of fluorite within the strata of the Kungurian beds.

Literary data on the fluorite findings in sedimentary rocks point in most cases just to this way of its formation. Thus, in a review by L. V. Pustovalov on ratovkite of the Upper Volga region ([3]), giving by the way a summary of all the necessary literary data, the genesis of fluorite and ratovkite in the Kashira beds of the Moscow basin is interpreted in a similar way. It would be of interest to note that the periodicity observed here in the fluorite-bearing carbonate rocks is reflected also in the mode of distribution of this mineral throughout the section: its precipitation is confined to the moments of maximum salinity. V. P. Baturin, who observed fluorite also in the Kungurian deposits of the Ural-Emba region ([4]), arrived at a similar conclusion. Some data on fluorite in carbonate-sulphate Permian rocks are given by

N. A. Uspensky ([5]), who observed fluorite in celestite concretions collected in the vicinity of the town of Kamensk (eastern slope of the Urals). As a matter for regret this author gives rather scant information on fluorite, consisting in some data on its spotty colouring and the approximate value of the refractive index. Of interest are similar data of G. I. Bushinsky on the genesis of fluorite ([6]). The possibility is not excluded that the fluorite segregations described by P. Krotov as early as in 1882 ([7]) for the Silurian limestones are of the same nature.

The same standpoint is confirmed by the physical-chemical investigations of A. V. Kazakov ([8]), who studied the equilibrium system in a number of combinations of different elements, including those of fluorine. Kazakov believes that the conditions of the medium, where the accumulation of fluorite can take place, are strictly defined by the main physical-chemical parameters. Thus, fluorite can accumulate neither in sea basins with a normal salinity, nor in fresh-water continental basins poor in fluorine and abounding in phosphorus. Its formation and segregation is possible chiefly in drying basins, where the concentration of fluorine salts is equal to or above 10 mg/l. In such a case fluorite is one of the earliest pegnitogene minerals followed by sulphates and other halogen formations, after a further increase in the concentration of salts in sea water has taken place; this determines the sequence of their crystallization.

The views of A. V. Kazakov seem to us correct and in agreement with the physical-chemical conditions, under which the accumulation of the Kungurian beds must have taken place. Considering this, as well as the fact that mineral associations are regular for definite geochemical facies ([9]), it may be supposed that fluorite is much more widely spread in sedimentary rocks, in particular in Kungurian beds, than it has been known heretofore. In particular, it may be supposed that fluorite (as well as celestite) must be present in all cases when the dolomite sediments are replaced by sulphatolytes, in correspondence with a gradual increase both in the concentration of the salts and the pH value. The area of predominant fluorite accumulation will, evidently, be located here in the upper portion of the dolomite, probably in the part of the stratum underlying anhydrite and gypsum. Hence, it is quite clear that the finding of fluorite is most probable in the case of a joint occurrence of both rock types, though this does not exclude the possibility of findings of this mineral in purely dolomitic rocks, as it is observed in the Kashira beds.

From the above standpoint, the finding of fluorite in the section of each series of the Kungurian beds of Tuimaza becomes quite comprehensible. The maximum fluorite content is confined just to the portions of the dolomite bands underlying the anhydrite-dolomite rock and partly to its lower portions. Such an arrangement has been observed almost in all pockets and is by no means accidental, but, considering the above data, it is strictly regular also for other areas of distribution of analogous rocks. An evidence of such a place of fluorite segregation is its idiomorphism with regard to the sulphates and its xenomorphism with regard to the dolomite.

After the present paper had already been prepared, L. M. Miropolsky ([10]) published some new information on fluorite findings in the Kungurian deposits of Tataria. Fluorite was detected by him in a core from bore-hole No. 6, located at the right-hand bank of the Kama river near the village of Sokolyi Gory. The rock enclosing fluorite was pelitomorphous dolomite filled with gypsum concretions. Within the same area were also found chalcedony (probably lutecite), quartz and celestite. Fluorite forms grains frequently crystallized as a cube, usually colourless, though in places a faint pink colour may be observed. As may be seen from this description, the conditions under

which fluorite was found on the Kama river, closely approach those observed at Tuimaza. The information given by Miropolsky is interesting also by that it confirms the ideas previously advanced as to the extensive regional development of this mineral in the Kungurian, which seems to be in accord with its precipitation under definite conditions of salt concentration in the basin.

I. M. Gubkin Oil Institute.
Moscow.

Received
25. I. 1941.

REFERENCES

[1] В. П. Флоренский, Очерк литологии кунгурских пород Туймаз. нефт. м-ния в Башк. АССР, гл. I—XIV (1939). [2] В. П. Флоренский, Тр. Моск. нефт. ин-та, II (1940). [3] Л. В. Пустовалов, Тр. Ломон. ин-та Акад. Наук СССР (1937). [4] V. P. Baturin, C. R. Acad. Sci. URSS, XIX, No. 6—7 (1938). [5] И. А. Успенский, Тр. Ломон. ин-та Акад. Наук СССР, вып. 7 (1936). [6] G. I. Bushinsky, Bull. Acad. Sci. URSS, sér. géol., No. 5 (1936). [7] П. Кротов, Изв. Геол. ком., II (1883). [8] А. В. Казаков, Сов. геол., No. 6 (1936). [9] Л. В. Пустовалов, Петрография осадочных пород (1940). [10] L. M. Miropolsky, C. R. Acad. Sci. URSS, XXV, No. 6 (1939).

Translated by V. Gurian.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

APPLIED GEOPHYSICS

THE METHOD OF HORIZONTAL ANTENNA AS APPLIED TO THE MEASUREMENT OF THE GLACIAL COVER

By A. A. PETROWSKY and B. N. DOSTOVALOV

(Communicated by V. A. Obruchev, Member of the Academy, 23. I. 1941)

As early as 1929 the German physicist Stern had reported a wave length measurement method for measuring the thickness of ice (1). In this country no such experiments had been carried out, and since the measurement of the thickness of ice might be of great practical value, it was decided at the V. E. Obruchev Institute of Permanently Frozen Soil to reproduce them in order to make more precise and develop further the wave length measurement method. The experiments were carried out with the aid of the PA-5 apparatus (2) at the water station of the «Locomotive» sporting society (the town of Lyublino near Moscow) in the winter of 1939—1940. The depth of the lake at the place of operations was 170 cm. To do the necessary work, the ice was freed from the covering snow over an area sufficient for stretching the antenna and arranging the measuring apparatus (this cleared area represented approximately a rectangle with 12 m and 4 m sides). Over the cleared ground there was stretched a horizontal symmetrical antenna characterized by the following data: length of each wing $l = 497.0$ cm; distance between the wings $2p = 80$ cm; average radius of wire $r = 0.919$ mm.

The elevation of the antenna above the surface of the ice could be varied within a range of from 5 to 60 cm (usually the observations were made at 5 or 6 different heights). The measurement of the effective capacity of the antenna was made by tuning in resonance for a given length of the wave (3).

The change of the strength of current in the secondary circuit of the device upon substituting a condenser for the antenna constituted from 4 to 15% and practically exercised no influence upon the tuning. The observations were carried out by B. N. Dostovalov who measured in several places the thickness of the ice with a specially made gauge, yielding an accuracy of about 5 mm. every time before making an observation. A total of 6 series of observations of the ice were made. The first series was made on Nov. 27, 1939, the temperature of the air being —5° C and the thickness of the ice $d = 8.7$ cm; the latter series made on Jan. 28, 1940, at a temperature of the air —12° C and a thickness of the ice 49.7 cm. Moreover, since the spring of 1939 several observations above the water had been made, the apparatus and the supports of the antenna having been placed on the lower step of a bathing platform.

The results of observations are shown in Fig. 1, in which the effective capacity C_{ae} of the antenna is plotted against its elevation h. The curves

are numbered, and the thickness of the ice, corresponding to each curve, may be found in Table 1.

Table 1

No. of curves	1	2	3	4	5	6	7
Thickness of ice in cm	0	8.7	12.3	17.1	23.0	37.5	49.7

Table 2

No. of curves	1	2	3	4
Evaluation of antenna in cm . .	10	20	30	40

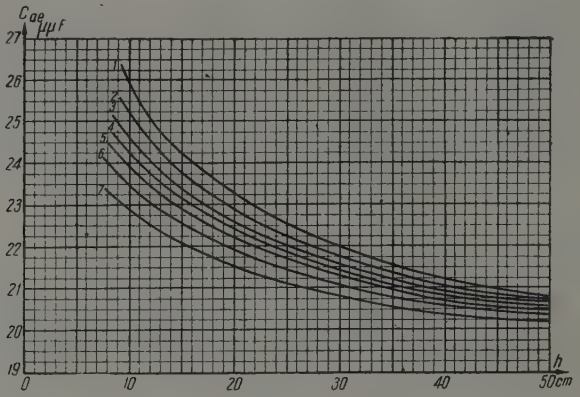


Fig. 1.

Fig. 1 shows that for all heights the effective capacity of the horizontal antenna markedly diminishes with an increase in the thickness of the ice. In order to make more evident the dependence of the capacity C_{ae} on the thickness of the ice d, a number of new curves shown in Fig. 2 were constructed on the basis of the curves of Fig. 1. Each of the curves of this graph corresponds to a definite elevation of the horizontal antenna (Table 2), the thickness of the ice being represented as a function of the measured capacity C_{ae}.

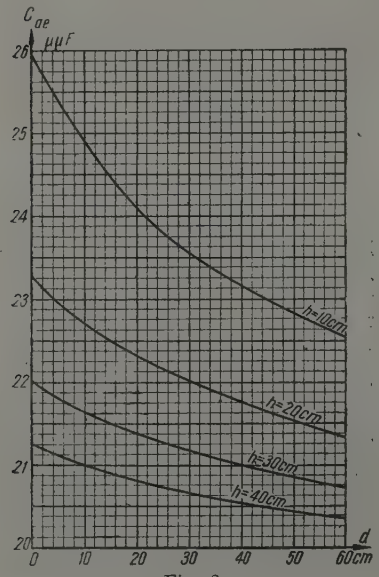


Fig. 2.

When such a graph is available, and when an observation of the value of the capacity of the horizontal symmetrical antenna, having the above characteristics, is made at a certain point of the ice surface, it is evidently possible to determine the thickness of the ice cover without drilling or in any way breaking its continuity; for this it is sufficient to draw in Fig. 2 an abscissa corresponding to the measured capacity C_{ae}, until it intersects the curve corresponding to the given height h. For instance, if with a height $h=20$ cm

a capacity of the antenna $C_{ae}=22$ $\mu\mu$F is obtained, the thickness of the ice is $d=30$ cm. The above results of the observations of capacity are laid by the Institute at the basis of a special design, named the «capacitative measurer of soil freezing». The purpose of this device is to determine the thickness of the frozen upper layer of ground without breaking its continuity. Such an apparatus will be of practical value in the first instance for roads, asphalt roads included, where in the frost raised places the data concerning the thickness of the frozen layer will furnish the necessary information for developing measures to be taken against heaving by frost.

On the same principle may be designed devices for settling many other practical questions, where conducting masses, invisible to the observer, are separated from him by a layer of a dielectric, or where there are several dielectrics with different dielectric constants.

Received
24. I. 1941.

REFERENCES

[1] W. Stern, Gerlands Beiträge z. Geophysik, **23**, H. 3, 292 (1929). [2] В. Алексеев, Тр. ИГН АН СССР, вып. 22, инж.-геол. серия, № 1, 21 (1940). [3] А. А. Петровский, там же, № 1, 1 (1940).

Translated by A. Brashnina.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

BIOCHEMIE

ÜBER DIE ABLAGERUNG VON FETT IN DER SCHALE VON VOGELEIERN

Von I. J. PRITZKER

(Vorgelegt von A. N. Bach, Mitglied der Akademie, den 17. I. 1941)

Auf der Suche nach biologischen Merkmalen, durch die sich die biologisch unzulängliche Vollwertigkeit von Eiern des Hausgeflügels bestimmen lässt, wandten wir unsere Aufmerksamkeit dem Umstand zu, dass Enteneier in vielen Fällen von einem besonderen Belag bedeckt sind, der den Eindruck von Fett macht. Die Anzahl der Eier, deren Schale mit einem solchen Fettbelag bedeckt ist, erreicht in einzelnen Wirtschaften 50% und mehr.

Zwecks Klarstellung der Natur dieses Belags wurde die oberflächliche Schicht solcher Schalen in Dicke von 0.02 mm abgeschabt und das erhaltene Material auf seinen Gehalt an rohem Fett und Protein untersucht. Zur Kontrolle wurde von Eiern ohne solchen fettigen Belag die Oberflächenschicht in gleicher Dicke abgeschabt und analysiert.

Die Analyse ergab, dass die Differenz in der Menge organischer Substanz in der Oberfläche fetter und trockener Eierschalen verhältnismässig gering ist. Aber die Zusammensetzung dieser Substanz weist beträchtliche Unterschiede auf: während in der Oberflächenschicht trockener Eierschalen das Eiweiss 9.70% und das Fett 6.53% ausmacht, beträgt in solchen mit fettigem Belag der Eiweissgehalt nur 2.65%, der Fettgehalt dagegen 11.90%. Von der Oberflächenschicht der fetten Schalen war das Eiweiss zu 70% von Fett verdrängt, das mehr als 80% der Gesamtmenge an organischer Substanz in dieser Schicht ausmachte.

Die Analyse der ganzen Eierschalen mit Fettbelag zeigte, dass ihr Prozentgehalt an Eiweiss um 1.5 Mal, an Fett um 3 Mal höher ist als bei trockenen Eierschalen. Interessant ist der Umstand, dass bei den ganzen Eierschalen keine Verdrängung des Eiweisses durch Fett zu beobachten war, wie es in der Oberflächenschicht der Fall ist, sondern dass eine parallel verlaufende Vermehrung des Fett- und Eiweissgehalts vorliegt. Infolgedessen enthalten die Schalen mit Fettbelag doppelt soviel organische Substanz (11.57%) als trockene Schalen (5.89%).

Um klarzustellen, ob derartige Enteneier mit erhöhtem Fettgehalt der Schale biologisch vollwertig sind, nahmen wir Brutversuche im Jamesvy'schen Inkubator mit neun Sätzen von Enteneiern vor. Es wurden hierzu Eier von möglichst regelmässiger Form, von Standardgewicht und im Alter von 3—5 Tagen ausgewählt, die von Enten stammten, deren Pflege- und Fütterungsbedingungen die gleichen waren. Die Bretter mit den Eiern der Versuchs- und der Kontrollgruppen wurden paarweise in den Kammern

ein- und derselben Abteilung des Brutapparats untergebracht. Da die Versuche unter den Verhältnissen eines Geflügelzucht-Grossbetriebs angestellt wurden, waren die Schwankungen der Temperatur und der Luftfeuchtigkeit in den Kammern des Inkubators für Versuchs- und Kontrollgruppen von synchronem Charakter und konnten darum keine wesentlichen Unterschiede in den Brutresultaten bewirken.

Die Resultate des Rekognoszierungsversuchs (n=315) waren folgende: während die Inkubation von Eiern mit trockener Schale 56.1 % junge Entchen von der Gesamtzahl der bebrüteten Eier ergab, betrug das Brutresultat von der Gesamtzahl der Eier mit Fettbelag im ganzen nur 30.5%. Die angeführte Tabelle zeigt die Resultate von siebenmaliger Wiederholung des Versuchs.

	Zahl der Eier		Verendete Embryonen (in % der befruchteten Eier) nach Tagen				Ausgekrochene Entenküken			In % von der Zahl der Eier	
	unter Abzug d. zerbrochenen*	nicht befruchteten	1—7	8—14	15—21	vom 21 ab	insgesamt	kräftige	schwache	der bebrüteten	der befruchteten
Eier mit Fettbelag . . .	80	32.5	29.6	7.4	14.8	42.6	3	—	—	3.75	5.6
Ditto	94	35.1	29.0	6.6	16.4	34.4	12	66.6	33.4	12.9	19.6
»	132	35.5	37.8	11.9	14.1	15.2	18	38.3	66.7	13.3	19.6
»	138	41.3	23.5	11.1	11.1	29.6	20	40.0	60.0	14.5	24.7
»	137	25.6	1.0	13.7	35.3	34.3	16	12.5	87.5	11.7	15.7
»	137	28.5	6.1	10.2	16.3	32.6	34	58.8	41.2	24.8	34.7
»	282	26.2	8.6	16.3	8.1	36.5	63	6.4	93.6	22.4	30.7
Mittelwert . .	1.000	30.4	15.8	12.3	15.7	32.3	166	36.3	63.7	16.5	23.9
Eier ohne Fettbelag . . .	80	21.2	6.3	1.6	3.1	56.1	20	—	—	25.0	31.9
Ditto	92	21.7	9.7	1.3	15.3	6.9	48	79.2	20.8	52.2	66.8
»	134	32.8	7.6	3.3	2.2	31.1	50	60.0	40.0	37.9	55.7
»	138	23.9	5.7	2.9	6.7	44.7	42	52.3	47.7	30.4	40.0
»	137	13.9	0.9	5.1	11.9	25.4	67	74.7	25.3	48.9	56.7
»	128	21.1	3.0	4.0	8.0	34.6	51	76.5	23.5	38.9	50.0
»	282	23.8	4.6	5.1	2.3	30.0	125	68.0	32.0	44.3	58.0
Mittelwert . .	991	22.9	5.0	3.7	6.4	32.1	403	68.6	31.4	40.7	52.8

* Für die Versuche wurden Gruppen von je 1003 Eiern verwendet.

Bei Variationbearbeitung des Versuchsmaterials wird die Differenz in den Brutergebnissen noch anschaulicher (28.9 ± 4.14). Der Prozentsatz an schwächlichen und verkümmerten Entküken ist bei den Eiern mit Fettbelag gleichfalls ein höherer (39.3 ± 5.8). Die Differenz in dem Verhältnis zwischen befruchteten und unbefruchteten Eiern war keine beträchtliche und lag innerhalb der möglichen Fehlergrenzen (7.5 ± 3.84). Die Zahl der unbefruchteten Eier war für beide Versuchsgruppen eine hohe. Während der ersten drei Wochen der Inkubation betrug die Sterblichkeit der Embryonen in den Eiern mit Fettbelag das 2.5—3-fache von der Sterblichkeit der Embryonen in Eiern mit trockenen Schalen. Im Verlauf der letzten Brutwoche war die Zahl der in der Schale verendeten Embryonen für beide Versuchsgruppen ungefähr die gleiche.

Das Abspülen des Fettbelags von der Schale, wie wir es in einem unserer Versuche ($n=282\times3$) vornahmen, ergab keine Verbesserung der Brutresultate. Während aus den Eiern ohne Fettbelag 58% Entchen ausgebrütet wurden, betrug der Prozentsatz lebend auskriechender Entchen aus den Eiern mit Fettbelag und solchen, von denen dieser Belag abgespült worden war, insgesamt 30.7 und 32.3%.

Wenn wir die Mortalitätskurve der Embryonen betrachten, die aus Eiern von nicht richtig ernährten Entenherden stammen, ferner die Tatsache, dass das Brutergebnis sich nicht erhöht, wenn man den Fettbelag von den Schalen abwäscht, und schliesslich den hohen Prozentsatz an nichtbefruchteten Eiern, so scheint die Annahme gestattet, dass der Fettbelag auf der Schale von Enteneiern und der erhöhte Prozentgehalt an organischer Substanz in der Gesamtmasse der Schale nicht so sehr die Ursache für die hohe Sterblichkeit der Entenembryone darstellen, als vielmehr eine merkmal wesentliche Veränderung im Plasma der Eier. Man kann kaum annehmen, dass nur die verhältnismässig indifferente Schale Veränderungen unterworfen ist und der Inhalt keinerlei Abweichungen von der Norm erleidet. Letztere Vermutung bedarf jedoch der Bestätigung durch spezielle Analysen.

Das Vorhandensein eines hohen Prozentsatzes an Eiern mit Fettbelag kann demnach als ein diagnostisches Merkmal betrachtet werden, nach dem man herabgesetzte Fortpflanzungsfähigkeit konkreter Gruppen von Enten im Laufe eines konkreten Zeitintervalls beurteilen kann.

Baschkirisches Landwirtschaftliches Institut.
Ufa.

Eingegangen
den 21. XII. 1940.

Übersetzt von A. Bernfeld.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

GENETICS

POLYPLOIDS IN *PAPAVER SOMNIFERUM* L. INDUCED BY TREATMENT WITH COLCHICINE

By E. N. VOLOTOV

(Communicated by A. A. Richter, Member of the Academy, 24. V. 1940)

In the present paper the author describes the production of polyploids in *Papaver somniferum* L. and his observations on their development during one summer.

Cultivated varieties of opium (Chinese, Tian-Shian, Tarbagatai, Turkish) and oil poppy (Tatarstan) were treated with colchicine. The material was not inbred, so as to make more room for subsequent selection. Both seeds and growing points were treated with 0.2% solutions. All varieties tested did not respond to colchicine in the same way. In the case of opium poppy the germination was 15% after one day of treatment, 5% after 2 days and 0.5% after 3 days. In the case of oil poppy these figures were 11, 3 and 0%, respectively.

At the beginning of their development the experimental plants had short, thick stems and leaves. Plants, the most changed morphologically, frequently perished at various stages of development, especially, at the cotyledon stage. The surviving plants gradually altered their habitus, and at the time just before flowering very few of them differed from the control.

Pollen and stomata of the treated plants were investigated for polyploids. Among the 420 plants subjected to one day treatment a plant containing mixed pollen (diploid and tetraploid) was found. Among the plants treated for 2 days 53 plants with chimeral and 17 with homogeneous pollen were found (in all, 70 plants). Among the 53 plants with mixed pollen 4 plants contained different grades of diploid, triploid, tetraploid and even octoploid pollen. The 17 plants with homogeneous pollen consisted of 16 plants with diploid and 1 with tetraploid pollen. Treatment for 3 days usually resulted in sterility and destruction of the plants. One plant with mixed pollen was found among 35 plants that had survived and given pollen.

The relatively low per cent of polyploid plants obtained (in the experiments with 2 day treatment, 3.2%) might be explained by unfavourable meteorological conditions (drought in the summer of 1938) and by the capacity of poppies to lose the polyploid cells in the chimeras. The treatment for 3 days gave the best results under these conditions.

The stomata were investigated in 316 plants chosen owing to their markedly altered habitus. This method proved to be less accurate.

The growing point was treated twice, each time for 12 hours with an interval of 5 days. The colchicine was applied at the stage of 5—6 leaves

by means of a cotton sponge imbibed with 0.2% solution. A total of 112 plants were so treated; 26 plants of this number had chimeral and 1 plant had purely diploid pollen. The mixed pollen consisted of a high per cent of tetra-, octo- and even hexadecaploid pollen. The results obtained stress the advantages of this method as compared to the method of seed soaking.

The cytological investigation was carried out by means of the aceto-carmin method. An unjust contempt exists for this method. The author of the present paper has verified it on poppy, pea, tomato, oats, flax, vetch, etc., and has found it suitable not only for chromosome counts, but for the study of chromosome structures as well.

The preparations were made in the following way. The roots were fixed for 24 hours with Carnoy and were subsequently transferred in 96° alcohol. A growing point $^1/_2$ mm long was stained for 20—30 min. in acetic carmin on a glass slide and was subsequently squashed under a cover glass. A fresh drop of carmin was added, the preparation was heated almost up to boiling and was squashed again. Addition of carmin, heating and squashing were repeated once more. The preparation was kept embedded in paraffin for 2—3 days. Some one with experience can prepare about 100 preparations a day.

N. N. Sokolov of this Institute has worked out a simple and rapid method for transferring temporary preparations into permanent ones. This method has yielded good results. It is easy to count the number of metaphase and anaphase chromosomes on the preparations. The chromomeres are also well visible and may be counted. Some chromosome parts stain more intensely, this indicates apparently the different staining capacity of the inert and active regions (9). All the chromosomes may be therefore individualized. Rapid obtaining of preparations makes this method irreplaceable for many cytogenetic and selectional works.

A cytological investigation of seed roots taken from 56 plants (5—10 seeds in each case) showed that in the majority of cases chimeral plants were obtained ($2n=44$) bearing tetraploid and diploid seeds in various proportions. Fig. 1 illustrates a metaphase from the root of a diploid poppy; Fig. 2, a metaphase of a tetraploid poppy. So far tetraploid seeds have been found in the progeny of 14 plants, but purely tetraploid seeds have been found in two cases only. The tetraploid seeds differed markedly from the diploids by their size, irregular shape and wrinkles. Large tetraploid seeds were retained in the chimeral plants located in the same capsule; they differed markedly from the diploids. This fact refutes the assumption of Randolf (7) that the size of tetraploid seeds in maize is due to bottom nutrition, because the tetraploid cobs are not well filled with seeds. However, the tetraploid seeds germinated normally. In the future the characteristic shape and size of the tetraploid seeds will facilitate finding them in the chimeral plants, where so far they have not been detected cytologically. Triploid seeds have not been found at all. This indicates the difficulty encountered in crossing tetraploid and diploid forms, this fact having been proved also experimentally.

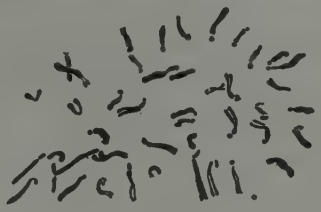
The pure tetraploids gave plants with thick stem and leaves, large flowers and a large flattened capsule, and were more pubescent than the diploids. Their vegetation period lasted from 6 to 8 days longer. Some flowers had an altered habitus, they gave etiolated leaves and perished at various stages of development. This may be explained by variations in chromosome number, since a later investigation of the progeny of tetraploid poppies has established one deviation with 42 chromosome and one with 38 chromosomes among 71 seeds examined. The habitus of the tetraploid poppies was so evident, that it could easily be discerned among the progeny of the chimeras. The following kinds of tetraploid forms are available at the present time: Chinese, Tarbagatai, Tian-Shian and Tatarstan. The first two kinds

have been investigated cytologically, the two second have been picked up from the progeny of chimeras owing to their different habitus. Besides, some tetraploids have been produced by treatment with colchicine from F_1 of an interspecific cross between Tatarstan (oil) and Chinese (opium) poppies.

In tetraploids the seed number was 6 to 10 times lower than in the control. This quantity suffices for cultivating opium poppies, whereas for oil poppies it is insufficient. The problem of how to increase the fertility of tetraploids referes not only to poppies, but to autotetraploids of other plants as well. Certain theoretical premises make the successful solution of this problem very likely, and it will become possible then to use the positive qualities of autotetraploids for practical purposes.

A total of 58 crosses were carried out in order to check the crossing capacity between diploid and tetraploid forms. Among this number 7 crosses

Fig. 1.



F.g. 2.

between diploids and tetraploids took place. The seeds were weak and smaller than those of the diploids, their number ranging from 2 to 14. The cytological investigation of 3 seed roots confirmed their triploid nature. The results obtained by this and by several other authors ([10]) indicate that this new form is isolated from the initial one.

The description of tetraploid poppies given above draws a clear-cut boundary between them and natural tetraploid species of *Papaver setigerum* D. C. which has 44 somatic chromosomes and which, according to some authors ([11]), has the same ancestor as *Papaver somniferum* L. Wild forms of *Papaver setigerum* L. are widespread throughout the Mediterranean (south of France, west of Italy, Sardinia, Morocco, Algiers). These forms have a short habit, small capsule and minute seeds.

Prolonged selection of both opium poppy and oil poppy and the accumulation of a great number of mutations have, no doubt, served as a basis for the differences existing between tetraploid *Papaver somniferum* L. and the above-mentioned species.

Institute of Experimental Biology.
Academy of Sciences of the USSR. Moscow.

Received
16. V. 1940.

REFERENCES

[1] A. Heilbronn, ZS. f. indukt. Abst. u. Vererb.-Lehre, Suppl. Bd II (1928). [2] D. Kostoff and I. Aksamitnaya, C. R. Acad. Sci. URSS, XI, No. 3—4 (1935). [3] Р. Н. Уринсон, Тр. по прикл. бот., ген. и сел., сер. III, No. 13 (1936). [4] F. W. Sansome and S. S. Zilva, Biochem. J., **30** (1936). [5] С. А. Щавинская, Тр. по прикл. бот., ген. и сел., сер. II, № 7 (1937). [6] M. L. Ruttle and B. R. Nebel, Biolog. Zbl., **59**, H. 1/2 (1939). [7] L. F. Randolph, Journ. of agric. research, **50**, 591, 605. [8] L. F. Randolph and D. B. Hand, Science, **87**, N. S., 442 (1938). [9] E. Heitz, ZS. indukt. Abst. u. Vererb.-Lehre, LXX, H. 3/4 (1935). [10] Г. Д. Карпеченко, Тр. по прикл. бот., ген. и сел., сер. II, No. 7 (1939). [11] М. А. Веселовская, там же, прил. 56 (1933).

Translated by C. Moisseenko.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

GENETICS

PRODUCTION OF TETRAPLOIDS IN *RICINUS COMMUNIS* TREATED WITH COLCHICINE

By B. N. SIDOROV and N. N. SOKOLOV

(Communicated by A. A. Richter, Member of the Academy, 24. V. 1940)

The rôle played by polyploidy in the evolution of plants is at the present time an established fact. The available investigations ([2], [3]) stress the significance of polyploid forms among the flora of the North. Due to these observations, the production of polyploids presents considerable interest from the evolutionary point of view and from the point of view of practical selection as well. If the available data on the greater adaptability of polyploids to northern latitudes are correct, the production of polyploid forms in southern plants will contribute to the expansion of these crops to the North. It is a known fact that polyploidy alters many characters of the organism and, first of all, the processes of accumulation of chemical substances. The production of polyploidy in *Ricinus communis* is therefore of some interest. *Ricinus communis* is a tropical perennial plant from the family of *Euphorbiaceae* and is cultivated as an oleiferous annual crop. In the USSR *Ricinus* is cultivated in the southern regions (Central Asia, North Caucasus, Ukraine).

The method of treatment with colchicine suggested by Blakeslee and Avery ([1]) was used for inducing tetraploids. The experiments were carried out in the summer of 1938. As test material we used seeds from plants brought to us from the Crimea by N. K. Koltzoff and from plants given us by the All-Union Institute of Aromatic and Medicinal Plants. We express our sincere gratitude to N. K. Koltzoff and to the Institute of Medicinal Plants for having supplied us with this material. The growing point of the plants was treated with colchicine solutions varying from 0.05 to 0.2% for periods from 1 to 4 days.

The results of the colchicine treatment became easily manifest due to the emergence of chimeral tissues consisting of a mixture of diploid and polyploid cells, in the shoots and leaves which developed from the colchicine treated growing points.

The pollen from the aberrated shoots was analyzed and the results are shown in the table.

It is noteworthy that along with diploid pollen large pollen (apparently tetraploid) and small abortive pollen were frequently found. The plants which had shown some amount of diploid pollen were fertilized artificially with pollen from plants which had the highest per cent of large pollen. Part

Large, diploid pollen in %	0	0—1	1—5	5—10	10—20	Above 20	Total of plants with aberrated pollen
% of plants manifesting the respective quantities of aberrated pollen	57.6	10.2	11.8	8.4	5.1	6.9	42.4

of the seeds obtained in this experiment were grown and the roots were studied cytologically.

Seeds were analyzed from 27 plants treated with colchicine, and among them 3 plants were found with tetraploid seeds. The seeds from three branches were examined. In the plant No. 158 one of these branches gave only diploid seeds (10 seeds), the second branch had one tetraploid seed, and the third branch had only tetraploid seeds (8 seeds). In plant No. 88 the seeds were examined from three branches. Two branches gave diploid seeds, and one branch gave one tetraploid seed. In plant No. 164 the seeds from two branches were examined. One of these branches was normal, the second branch was chimeral and exhibited along with a great quantity of diploid seeds two tetraploids.

We examined a total of 420 seeds and not a single triploid was found, although in the majority of cases we used for fertilization pollen from plants with not less than 50% of diploid pollen. Normal diploid plants of *Ricinus* have 20 chromosomes. All the newly obtained tetraploids had 40 chromosomes.

A preliminary analysis showed that the newly produced tetraploids of *Ricinus* differ morphologically from the diploids. The developing leaves have a dark-green coloration, they are thicker and less cleaved. The tetraploids have larger cells. In the tetraploids the stomata are also larger than the diploids. The average dimensions of the stomata in the diploids equals 7.5 grades of the ocular micrometer, in the tetraploids it amounts to 9.10. The pollen grain of the tetraploids also differs from that of the diploids. The average dimensions (diameter) of a pollen grain of the tetraploids equals 29.5 grades of the ocular micrometer, as against 24.4 grades of the haploid pollen. Along with normal grain the pollen of the tetraploid plants contains a certain quantity of abortive pollen varying in different plants. The seeds from tetraploid plants surpass those of the diploids almost by 1.5 times in respect to dimensions and weight. The absence of triploids established by us in the experiments on inducing polyploidy in the summer of 1938 indicates the absence of fertilization in the haploid germ-cells of the diploid pollen, and vice versa. In order to check this fact, special experiments on the fertilization of diploid plants with pollen from tetraploids and vice versa were carried out in the summer of 1939. No progeny was obtained in the numerous crossings. Thus, we have evidence that the newly induced tetraploid form of *Ricinus* cannot be crossed with the initial diploid form of *Ricinus*.

Institute of Experimental Biology.
Academy of Sciences of the USSR.
Moscow.

Received
16. V. 1940.

REFERENCES

[1] A. F. Blakeslee and Avery, Journ. of Heredity, **28**, 393 (1937) [2] G. Tischler, Engler's Botan. Jahrb., **67** (1935). [3] H. D. Wulf, Jahrb. f. wiss. Botan., **48**, 5 (1937).

Translated by C. Moisseenko.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS.
1941. Volume XXXI, № 3

GENETICS

MUTATIONS RESTITUTING THE TELOMERE

By J. A. RAPOPORT

(Communicated by A. A. Richter, Member of the Academy, 6. II. 1941)

The analysis of attached chromosomes bearing differently marked extreme left ends gave a genetical proof of the formation of telomereless fragments of the sex-chromosome, capable of reproduction ([5]). A cytological study of the stable fragment structures found in these experiments and in the male progeny of X-rayed *XX* females without a sex-chromosome has shown, that their microscopic appearance varies from rod-shaped, like that of a normal chromosome, to double-armed structures, whose longer arm is not shorter than the short arm of the *Y*-chromosome. Various intermediate types are found between these two extremes. Therefore, fragmentation is a method which may be utilized in producing hyperploidy for a desired region of the proximal part of the *X*-chromosome.

The fact that the genetic limit of such fragments does not go in males as far as the 56th morganide, is proved by a special mark on one of the arms of the *XX*, namely a lethal from the so-called *ClB* line. The males which received the arm with *ClB* and the fragment of the other arm of *XX*, reaching as far as the 56th morganide and capable of suppressing the action of a lethal, do not survive, owing to hyperploidy. In females which are capable of supporting with or without reduced fertility a much higher hyperploidy, the fragmented arm surpasses the boundary length of the corresponding block in males. Although the breaks of *XX* are confined to the proximal inert half of a chromosome, there is no ground to assume that the latter is more subject to breaks, since the fragments surviving in the female have often been found to be broken in the active region.

The presence of the rod-shaped fragments, closely reminding a normal *X*-chromosome, gave rise to the assumption that they have been formed as a result of fragmentation of the centromere itself. As the centromere apparatus is very complicated in some species, e. g. in *Ascaris* and in some insects, and the possibility of a transversal reduplication of the centromere has so far not been analyzed genetically, purely cytological data on this question being not sufficient, a special experiment was carried out. In order to decide whether the centromere of chromosome III in *D. melanogaster* is capable of undergoing fragmentation and to give a kinetic body to each of the linkage groups IIIL and IIIR, which must in this case form from the third chromosome, the following method was applied. Wild type females were X-rayed (3000 r) and crossed with males of the *ru h th st cu sr*, males homozygous for the translocation III—IV with a break of chromosome III

in the inert region. As irradiation objects were chosen females instead of males in order to observe the fragmentation process with a minimum admixture of other kinds of chromosome rearrangements. The heterozygotes were crossed with *ru h th st eu sr es ca* females free from translocation, and when new attachment groups were formed through the fragmentation of the centromere, the absence of attachment between IIIL and IIIR could be established.

This attempt, which has not yet been made with any other object, to obtain in this way a new attachment group, gave the following results. Among 2411 individual cultures only two showed absence of attachment between IIIL and IIIR. In one of them it was caused by a translocation between chromosomes II and III with a break of III in the inert region. The analysis of the other is not yet finished. A comparison of these data with the high percentage of breakages in *XX* after irradiation with 3000 r clearly shows that fragmentation of the centromere has nothing to do with them, and that the rod-shaped fragments of attached *X* are the result of a break in the subcentral region.

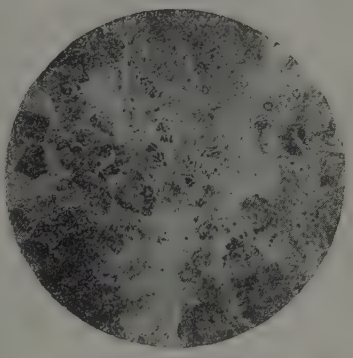


Fig. 1. Anaphases in a stock bearing a fragmented sex-chromosome. Bridges are absent.

So far the restitution of such a complicated formation as the centromere has not been observed; in our previous work, however, we have described restitution phenomena for the telomere, another normal chromosome organella. Part of the males with fragmented chromosomes are normally viable and fertile already in F_1, i. e. about ten cell divisions after the break. It is obvious that either a reparation of the defect in one of the first divisions of the somatic and germ cells took place, or else a break of the periodically fusing «bare» ends of the daughter chromatids occurred between the kinetic bodies of each of them, without disturbing the central apparatus. No departures from the structure of organs and tissues, inevitable after chance breakages of fused sister threads, have been observed. Microscopic examination in the following generations has shown reparation of the defect and de novo formation of an organella, corresponding to the telomere. Thus, the second hypothesis was discarded. A study of the anaphases in F_3 and F_4 of the three lines bearing fragmented chromosomes showed the absence of bridges (Fig. 1), whereas the *X*-chromosomes were unitypical at metaphases in each of these stocks. Consequently, there was no fusion. This is confirmed by the data on sex ratio in such lines, non-disjunction, etc., not differing from the control.

Basing on the analysis of fragments, we assume that the restitution of the telomere takes place after breaks both in the heterochromatin and euchromatin regions, though it is necessary to take into consideration the presence of interstitial inert regions within the latter. Breaks occur with about the same frequency per unit of metaphase chromosome length in both these regions ([1]). Therefore, the telomere is a replaceable structure. Its substitution is a peculiar mutation, not connected or rarely connected with the occurrence of new visible characters, but restituting normal mitosis. This mutation is a leap-like irreversible blocking of active groups, layed «bare» by the fragmentation and apparently universal along the whole chromosome length, as a result of attachment of some kind of chemical groups

from the protoplasm. Judging by the relative duration of this process of restitution, the number of groups is limited. These terminal reparative mutations, as we should like to term them, do not follow the break immediately. Contrary to the opinion widespread in literature, we believe that other kinds of mutation do not appear directly after irradiation of the cell either, but not until several hours after the latter. In order to accurately determine the time necessary for the union of fragments, the following experiment was carried out. *Bar* males and fertilized *yy* females were X-rayed (2000 r). The progeny was examined for new attached complexes with *yellow* in one arm and with *Bar* in the other. Their occurrence would prove that the fragments in both germ cells may remain free till the formation of synkarion. Among the 92 females heterozygous for *Bar* no new attached systems were found. The time of the union of fragment in the male germ cells, where all the chromosomes are closely adjacent, is limited and occurs in the male pronucleus or even in the sperm.

Few data are available on the arrangement of the chromosomes in the sperm cell. The character of their arrangement in *Ancyracanthus* and *Filaria* is not indicative, since the sperms of the *Nematodes* differ from those of other animals. Bowen (2) in his experiments on the maceration of the sperms of *Hemiptera*, and Gowen and Gay on some indirect data on *Drosophila*, have come to the conclusion that the chromosomes are arranged in a linear order in the sperm. To verify this theory, we examined four kinds of preparations stained after Feulgen of the following objects: normal sperm cells, sperm cells without a sex-chromosome, with an extra sex-chromosome and, finally, with a closed *X*-chromosome. Neither data in favour of a linear end to end arrangement, nor in favour of the existence of a fixed localisation order of the chromosomes (especially in closed *X*) in the sperm cells were found. The site occupied by a chromosome is an accidental one. This is confirmed by a number of old studies on the histogenesis of sperm and by demonstrative pictures found by Champy in *Discoglossus* (3).

The question arises, whether the data obtained make it possible to insist on that fragmentation is invariably a primary process in the genesis of chromosome rearrangements, while the formations of inversions, translocations and of deletions are secondary processes resulting from chance union of blocks? In the literature very little attention has been given to the fact that the majority of data, sufficiently convincing from our point of view and speaking in favour of a contact mechanism of the origin of aberrations, is obtained from the study of rearrangements in a mature, specialized and peculiar animal germ cell. Data pointing to the importance of fragmentation as of an intermediate condition during the formation of rearrangements, were obtained by irradiating germ cells in plants and somatic cells. Our material speaking in favour of fragmentation is derived from the female germ cells of animals.

However, different results can be accounted for by the diversity of material. In the somatic cells, in female germ cells of animals and plants and even in the male gametes of plants, the arrangement of the chromosomes is diffuse, whereas in the spermatozoa the chromosomes are located on a very small space, and they doubtlessly come into contact and intertwist. For aberrations caused by contact processes which may occur not only in sperms, but during the genesis of spontaneous rearrangement in plants as well, the chances of being retained are very great. During these processes no chance combination of chromosome parts takes place, the new combinations bear the character of reciprocity, and disturbances of balance are rare. However, during fragmentation the blocks without the spindle fiber and the products of union of two spindle fiber attachments perish, while the other part of the fragments becomes restituted in the old order, and this, evidently,

explains the lagging of the number of rearrangements as compared with mutations caused by small doses of X-rays ([4,7]). Though fragmentation is the predominant mechanism of chromosome rearrangements under the action of X-rays in both sexes, in males contact processes are also of frequent occurrence. Experiments on the connexion of non-disjunction between non-homologous chromosomes have shown that ([6]) the cause of this lies in the fusion of chromosome matrixes. Sperm cells are irradiated with much higher doses, and the chromosomes in them are located much more compactly than in the eggs. Therefore, fusion which occurs in many places causes non-homologous exchanges, rather similar to crossing-over. In the cells of the eggs, owing to a rarer occurrence of sticking, a pure fragmentation process may sometimes be observed.

Institute of Cytology,
Histology and Embryology.
Academy of Sciences of the USSR. Moscow.

Received
10. II. 1941.

REFERENCES

[1] H. Bauer, M. Demerec and B. P. Kaufmann, Genetics, **23**, 610 (1938). [2] R. H. Bowen, Jrnl. of Morph., **37**, 79 (1922). [3] Ch. Champy, Arch. Zool Exper., **62**, 1 (1923). [4] C. P. Oliver, ZS. f. ind. Abst.-Vererb., 63 (1932). [5] J. A. Rapoport, C. R. Acad. Sci. URSS, XXIX, 612 (1940). [6] И. А. Рапопорт, Биол. журн., **7**, 661 (1938). [7] В. В. Хвостова и А. А. Гаврилова, там же, **4**, 905 (1935).

Translated by C. Moisseenko.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

GENETICS

ON ROENTGENOMUTATIONS IN REPETITIONS AND X-RAY MUTATIONS AS CONDITIONED BY BREAKS

By J. A. RAPOPORT

(Communicated by A. A. Richter, Member of the Academy, 6. II. 1941)

Bar belongs to the few genes whose mutability, both spontaneous and induced by X-rays, has been studied exhaustively. The frequent hereditary changes described by May ([4]) and Zeleny ([10]) in the *Bar* line have attracted the attention of investigators to this line. From the very beginning correctly, though in general terms, May explained these changes as the non-disjunction of a pair of factors. Later on Sturtevant has associated them with a new phenomenon of unequal crossing-over. The mutability of this gene was studied next by Morgan ([5]), Muller and Dippel ([6]) and others. Under conditions excluding unequal crossing-over (observed only in a few early studies of Zeleny), these authors failed to find any mutations, and a new spontaneous allelomorph was described only by Sturtevant. However, Hanson ([3]) obtained many hereditary changes under the action of X-rays, and this was confirmed later on by many investigators ([1], [2], [9]). The fourfold linear repeat is an advantageous object for the study of essential peculiarities of the mutation process. This structure can be applied to the solution of the problem, whether a mutation, i. e. a qualitative change of the gene, and not the position effect or a deficiency, affects one gene or a large chromosome block. The specificity of *quadruple-Bar* consists in that four identical allelomorphs are so closely located within one chromosome in this structure, that the ontogenetic function of each of them undergoes slight quantitative changes while they are in the repetition. Using this structure it is possible to show, whether the mutation is restricted to a single gene, or whether the simultaneous leap-like change may involve from two to four loci, and the specific chemical mutational factor is capable of irradiating along the chromosome.

In a normal chromosome where non-identical genes are located side by side, the absence of synchronous mutations of several genes can be ascribed to marked differences in the structure of the adjacent genes, which do not mutate in response to the same chemical mediators, rather than to the impossibility of simultaneous changes. Such an experiment is interesting from another point of view as well. It has been suggested ([7],[8]) that the divergence of genes is caused by changes through transgenation in one of the elements of multiple linear repetition. If so, one is mainly to expect the expression of dominant mutations, since a great number of *B* allelomorphs is capable of suppressing the recessive mutations in repeated regions.

The study of simultaneous mutations of equal neighbouring genes was started in 1936 by irradiating heterozygous $\frac{+}{+}$ B^q *Bx* females. The occurrence of fertile B^q males facilitated the problem. The offspring of the first fertile

y f B^q males were X-rayed. Owing to low fertility after the action of X-rays the results of the experiment lacked clearness. Several thousand of irradiated males crossed with *XX* females gave 630 sons without a single reversion. Therefore, in the future we used only *f* B^q *Bx* males which were far more fertile. Four sets of experiments were carried out in all; the results are shown in the table. In the principal experiments irradiated males were crossed with normal, not with *X*-attached females.

No. of sets	Dose in r	Number of flies examined	Reversion to		
			wild type	*B*	B^d
I	4 000	630	—	—	—
II	4 000	2 901	—	2	3
III	4 000	5 088	2	1	3
IV	4 000	3 411	1	—	4

Out of the 12030 flies three had wild type eyes, three had *B* and ten had eyes similar to B^d. One of the females with a complete reversion for *B*, which manifested no *Bx*, left no progeny. This change was possibly caused by a deficiency. Two other females manifested *Bx*, but one of them also proved sterile; the second one bore a lethal in the chromosome containing *f Bx*. When crossed with a *f* male the exaggeration for *f* was produced. Thus, it may be assumed, that the first and third reversions contained deficiencies for the entire B^q region. Among the three reversions to *B* one was caused by a dominant modifier from chromosome III. The second fly (sooner reminding of infra-*B* than of *B*) contained a lethal and a deficiency noticeable in the salivary gland chromosomes. Among the ten reversions to B^d two contained lethals, while in one case the change was caused by a modifier. Thus, no complete reversions from B^q to the wild type were found, which would not be connected with a deficiency or with reduced viability. Indeed, if chemical irradiations of a specific mutation product along the chromosome were to take place, and the mutation in one of the loci were to cause a similar process in the adjoining locus, such changes should have been found.

We found no new dominant mutations in the fourfold region, though we carefully analyzed all the dominant mutations appearing in this experiment. As was to be expected, the presence of a multiple linear repetition does not automatically lead to an increase in the mutational changes as compared with normal regions. The structure of the repetition and of its constituent genes does not give any basis for this possibility. However, this does not diminish the rôle of repetitions in the evolution of genes. In the historical process the divergence of hereditary important structures is caused by the mutation of reserve genes, which play no rôle in the normal development, but cannot be eliminated from the chromosome, where they are retained in various repetitions. Even restricted to the given normal mutability rate, these freely mutating «functionless» genes offer several mutational changes interwoven with other morphogenic processes, and can in case of sufficient complication give rise to completely new reactions.

The results of the experiments showing the absence of polygenic mutations, do not agree with the data of Dubinin and Volotov [2]. These authors studied mutations in the *double-B* stock. From cases when the complete reversion of *double-B* to the wild type was not followed by an aberration within the 16A region, they drew the conclusion as to the simultaneous mutation of two *Bar* genes. However, Volotov [1] has shown that the *Bar* change may be connected with remote rearrangements in the chromosome system, and we assume that the simultaneous changes of *double-B* are also caused rather

by the position effect than by mutation*. We believe that reverse B^d mutations are nothing else than position effect of maximum intensity. The complete reversion of B is no quantitative limit of the position effect of this gene. The position effect may completely suppress the action of two or even three, but not of four B. The monogenic nature of mutations in repeats, observed in the present work, is probable for the following reason. In the literature it is shown that chromosome breaks are frequently followed by mutations, and it is certain that breaks call forth some kinds of hereditary changes, i. e. the position effect.

Owing to the data available on telomere mutations, the connexion existing between breaks and mutations sensu strictu can be explained from a new point of view. In the above-mentioned work it is shown that the breaks laying bare the chemically active chromosome groups, are a condition necessary for leap-like changes, which take place from time to time, block these groups and restitute the peculiarities of a normal chromosome in the telomereless fragment. Judging by the frequency of the fusion of fragments, the «blocking» mutations are not numerous. In the majority of breaks the fragments do not lose their capacity to fuse, but sometimes in one, or far more rarely in both fragments, an irreversible qualitative change takes place. Without a temporary disturbance in the continuity of the thread, active groups taking place in this process are inaccessible or less accessible to chemical changes. Thus, a break is the essential condition for the occurrence of a mutation, it is not an accessory phenomenon. Even the normal position of genes after the occurrence of an X-ray mutation does not prove that no break of the thread, which uncovered a very limited region, was responsible for its origin, although mutations may apparently take place without breaks.

On the other hand, changes of the linear order of the genes do not give us the right to explain the hereditary change, without special investigations, as a position effect. A hereditary change may be due to chemical reconstruction, union or disintegration of uncovered active groups of hereditary important structures, which might have taken place in the course of temporary discontinuity of the chromonema, rather than to an unusual rearrangement of the fragments**.

Institute of Cytology, Histology
and Embryology.
Academy of Sciences of the USSR. Moscow.

Received
10. II. 1941.

REFERENCES

[1] Е. Н. Волотов, Биол. журн., **6**, 943 (1937). [2] Н. П. Дубинин и Е. Н. Волотов, Журн. общ. биол., **1** (1940). [3] F. B. Hanson, Science, **67**, 562 (1928). [4] H. G. May, Biol. Bull., **33**, 361 (1917). [5] T. H. Morgan, Carn. Inst. Publ., **399**, 201 (1929). [6] H. J. Muller and A. Dippel, Br. Jrnl. Exp. Biol., **3**, 85 (1926). [7] И. А. Рапопорт, Бюлл. экспер. биол. и мед., **2**, 258 (1936). [8] И. А. Рапопорт, Журн. общ. биол., **1** (1940). [9] Н. О. Шапиро, Биол. журн., **6**, 837 (1937). [10] Ch. Zeleny, Jrnl. Gen. Phys., **2**, 69 (1919).

Translated by C. Moisseenko.

---

* The following experiment shows that the simultaneous change of two genes is not connected with a plasmatic factor common to the entire cell. Attached X females homozygous for B were X-rayed. Among 6708 females not a single wild-type fly was found, though 6 heterozygotes were present. Thus, simultaneous mutation of two *Bar* does not take place when the genes are in different chromosomes.

** Several non-hereditary changes were found. In six cases the occiput was absent and the head was very narrow. In five cases the labial feelers were lacking. In two cases there were large white swellings in place of the eyes. These swellings were probably caused by enlarged optical ganglia as compared with those typical of B. A body colour darker than in the wild type should also be noted. This is probably caused by the pleiotropic action of the gene.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

PLANT PHYSIOLOGY

ONTOGENETIC DEVELOPMENT AND YIELD OF COTTON AS INFLUENCED BY THE PROPORTION OF MINERAL ELEMENTS IN THE NUTRITION

By A. A. KUZMENKO, L. L. SEREDENKO and V. A. GOIKO

(Communicated by A. N. Bach, Member of the Academy, 28. I. 1941)

At different periods of its life cotton, like other plants [1], puts different demands to the medium in which it develops. However, workers in the physiology of mineral nutrition of this plant (whose number was not very strong-after all) have treated the subject without reference to developmental period [2–12]. They raised their plants on solutions of unvarying composition. It is only quite recently that in the USSR a few experiments were reported, in which the nutritional requirement of cotton had been studied at different periods of development [13, 14].

Similar experiments were started also at the Section for Chemical Plant Physiology of this Institute. The development in Ukraine of the cotton growing industry called for such work to be done. In these experiments growth development and yield of the cotton plant were studied in relation to the influence of mineral elements supplied to it at different times. They were conducted in 1939 and 1940. Those of 1939 were run with water cultures, the nutrient solution of Hellriegel being made use of. The object was to find out those phases in the development of the plant when it had to be supplied with highest quantities of the respective nutrient elements, in order to give the greatest yield of raw cotton. Variety 1306, the most widespread in Ukraine, was selected for the tests. Its seeds were supplied by the Cotton experiment station of Ukraine (Kherson).

The whole length of the growing period was arbitrarily divided by us into four phases: 1) appearance of the 4th leaf; 2) outset of budding; 3) outset of flowering; 4) start of fruit-formation. At the beginning of the respective phase some of the plants tested were transferred from the normal solution of Hellriegel to one, in which the amount of one, two or all of the three nutrient elements had been increased 4 times. When that phase came to an end, the plants were taken back to the normal solution and grown on it until the day of harvest.

It has been found in that way that their need in nutrient elements for best growth and development was the highest at the time when they had just formed 3 or 4 leaves. This was reflected both in the yield of raw product and in the weight of different organs. Thus, the total weight per plant was 15.73 g, the weight of raw cotton 3.30 g, when an N : P : K ratio of 4 : 4 : 1

had been given at the first phase of development. But if the same solution had been supplied during the second phase, the yield was only 9.51 and 1.96 g, respectively. The plants given a normal solution of Hellriegel throughout the growing season gave a total yield of 8.42 g, and a row cotton yield of 1.43 g.

That the need of cotton for nutrient elements is at a maximum in the early phases of its development, has been proved also by M. Abutalibov on other cotton varieties ([13]).

With increased rates of nitrogen, phosphorus and potassium in Hellriegel's solution for the period between the appearance of the 4th leaf and the outset of budding, the development of cotton plants is complete 7 days earlier.

As the most remarkable results had been achieved with this set of plants, we decided to make it the basic set in our further tests which were started in 1940. These tests were made in water cultures at Kiev. Their results are partly shown in Table 1.

Table 1

Development and Yield of Cotton Plants, Variety 1306, with Different N:P:K Relations in the Nutrient Solution

N : P : K proportion		Duration of the growing period in days	Number of bolls per plant	Dry weight of raw cotton per plant in g		
formation from 3 to 4 leaves until flowering	from the outset of flowering until ripening			before frost	after frost	total
1 : 1 : 1	1 : 1 : 1	155	2.0	0.72	3.4	4.12
1 : 4 : 1	1 : 4 : 1	131	3.33	1.38	4.3	5.68
1 : 4 : 1	4 : 1 : 4	141	3.7	4.22	7.7	11.92
1 : 1 : 4	1 : 1 : 4	136	4.5	2.0	6.0	8.0
4 : 1 : 1	4 : 1 : 1	149	3.0	1.1	5.4	6.5
4 : 4 : 1	4 : 4 : 1	—	3.2	2.2	4.9	7.1
4 : 4 : 4	1 : 1 : 1	131	4.5	4.7	4.4	9.1
4 : 4 : 4	$\frac{1}{4} : \frac{1}{4} : \frac{1}{4}$	131	4.25	5.9	3.8	9.7

From these figures it is obvious that the development and yield of the plants tested were greatly affected by the N : P : K relation and by the time when these nutrient elements had been supplied. The maximum yield of raw cotton was obtained when the proportion of N : P : K in Hellriegel's solution had been 1 : 4 : 1 before flowering and 4 : 1 : 4 after flowering.

Two more sets, in which the yield per plant was remarkably increased and development accelerated, are those in which the doses of N, P and K were 4 : 4 : 4 before flowering, and were then reduced either to 1 : 1 : 1 or to $^1/_4 : ^1/_4 : ^1/_4$, respectively. In the first case the plants after flowering were transferred to the normal solution of Hellriegel for the rest of the season, but in the second they were given intermittently—10 days a normal solution of Hellriegel with reduced rates of N : P : K ($^1/_4 : ^1/_4 : ^1/_4$) and another 10 days the same solution with increased rates (4 : 4 : 4). In both sets the plants were 24 days ahead of the controls in full ripening.

This water culture experiment at Kiev was run parallel with a soil culture experiment, which took place in the southern cotton growing part of Ukraine. The latter was intended to furnish new data for further development of our main conclusions drawn from the results obtained in 1939. This was a pot experiment and it was carried out according to plan by V. A. Goiko under the guidance of the Plant Physiology Section of this Institute. When the 3rd or 4th leaf had appeared on the plants, the fertili-

zers were supplied in solution through a pipe and partly by pouring over. The ratio was 0.28 g of acting substance (ammonium nitrate, superphosphate and potassium chloride) to 1 kg of soil. The main results of this experiment are presented in Table 2.

Table 2

Growth, Development and Yield of the Cotton, Variety 1306, as Influenced by the Proportion of Nutrient Elements

N : P : K proportion	Length of growing in days	Height of plants at the time of harvest	Number of bolls per plant	Yield by one plant in g		
				all parts	raw cotton	seed
Without additional nutrient (control) . . .	183	52.6	1.3	11.23	2.27	1.49
1 : 1 : 4	161	91.9	2.4	33.30	5.27	3.47
4 : 4 : 1	149	92.4	6.1	58.46	9.59	6.25
4 : 1 : 1	149	93.7	3.9	51.18	7.80	5.06
1 : 4 : 1	165	86.3	2.5	35.98	5.95	3.86
4 : 1 : 4	147	96.9	3.7	51.30	6.83	4.44
1 : 4 : 4	163	85.5	3.5	38.21	8.06	5.13
1 : 1 : 1	157	73.6	2.6	30.39	5·54	3.59
4 : 4 : 4	145	92.9	7.9	65.55	16.23	10.43

As seen from these figures, the plants of the last set, supplied with extra nutriment in relation 4 : 4 : 4 while in the phase of 3 or 4 leaves, were 38 days ahead of the controls in development. And acceleration of development is a matter of utmost importance for growing cotton as far North as in Ukraine. It is the plants of the last set, too, that gave the highest yield both in regard to the raw product and as a whole.

We arrive therefore at the conclusion that an intelligent control of mineral supply to the cotton can not only secure higher yields of raw product, but is also a means to accelerate development.

Section of Chemical Plant Physiology.
Academy of Sciences of the Ukrainian SSR.
Kiev.

Received
29. I. 1941.

REFERENCES

[1] Т. Д. Лысенко, Теоретич. основы яровизации (1936). [2] H. Harper and Murphy, Soil Sci., **26** (1928). [3] R. Bartholomew and G. Jannsen, Jrnl. Amer. Soc. Agr., **20**, 10 (1928). [4] А. В. Благовещенский и сотр., Тр. Ср.-Аз. гос. ун-та, сер. VIII в, вып. 4 и 5 (1929). [5] В. Н. Любименко и сотр., Тр. Укр. ин-та прикл. бот., I (1930). [6] Д. А. Сабинин и сотр., Тр. Всес. Н.-и. ин-та по хлопководству, вып. 37 (1931). [7] О. Ф. Туева, Ср.-Аз. н.-и. ин-т по хлопководству, вып. 2, Химизация культ. хлопчатника, ч. 1 (1933). [8] О. Ф. Туева, там же, 48 (1933). [9] E. Phillis and T. Mason, Ann. Bot., III, No. 11 (1939). [10] E. Phillis and T. Mason, ibid., III, No. 12 (1939). [11] E. Maskell and T. Mason, ibid., **44** (1930). [12] E. Maskell and T. Mason, ibid., **45** (1931). [13] М. Г. Абуталибов, Тр. Бот. ин-та Азерб. фил. АН СССР, VI (1939). [14] В. И. Цивинский, Сб. раб. по биол. и физиол. хлопчатника (1939).

Translated by D. Rakhmanov.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

PLANT PHYSIOLOGY

INFLUENCE OF ILLUMINATION INTENSITY UPON THE RESPONSE OF PEANUTS TO DAYLENGTH

By A. I. CHELIADINOVA

(Communicated by A. A. Richter, Member of the Academy, 9. I. 1941)

The species of *Arachis* dealt with in this paper is *Arachis hypogaea* L. Although unknown in a wild condition, it is usually supposed to be a native of Brazil on the ground that several wild growing species of peanuts have been encountered there. Many of these species are found in forests, and so it may be assumed that the cultivated form of peanuts is better fitted to a reduced intensity of illumination, both with regard to storage of dry matter and to its reaction to daylength. In order to check these assumptions I started, along with some experiments on the reaction of this plant to photoperiod under natural daylight, similar experiments under reduced intensity of illumination. The tests were carried out at Krasnodar in 1937.

Two peanut varieties were involved: Spanish 0344 and Virginia Bunch. The seeds were planted May 27 into Wagner pots; 10 days after germination the test plants grown under reduced intensity of illumination on various photoperiods were placed into special chambers, the walls of which were made of a framework covered with gauze. Curtailing the daylength was brought about by setting the chambers up into additional frames, over which a light-tight oil-cloth was stretched. The control plants were allowed to develop under natural daylight and daylength throughout the growing period.

Even at early phases of development the experimental plants were easily distinguishable from the controls. They were taller and bore a larger leaf area. The short-day plants were lower than those receiving normal day under the same conditions of screening. In all sets flowering set in almost simultaneously, except for the late variety Virginia Bunch, whose short-day plants came into flower earlier and were bearing more flowers during the first period of flowering. However, the total number of flowers in these plants was lower than in those of the other sets, because their period of flowering was reduced. The same was observed in the case of the earlier variety, Spanish 0344.

The control plants had two maximums of flowering: the first by the end of July, and the second, by the end of August. But the test plants had only one maximum of flowering and this fell between the end of July and the beginning of August. Afterwards the intensity of flowering lowered. At that time flowering came to an end in the short-day plants, whereas in the test plants grown on natural daylength, flowering continued, although at a very low rate.

Towards the end of the growing period the control plants of both varieties had twice, and sometimes thrice, the number of flowers of the test plants. It was only natural to expect that the yield of beans in the former would be correspondingly higher than in the latter. As a matter of fact, after the plants had been harvested late in September, the relation was found to be somewhat different.

It may be seen from the figures referred to in Table 1 that the experimental plants, especially those on short photoperiods, bore a far smaller number of underdeveloped beans and genophores than did the control plants. This was because flowering, as stated above, had been complete by far earlier in the test plants than in controls, and the fruits set had sufficient time to come to full size. Another reason was that owing to early flowering these plants were provided with sufficient quantities of nutrient substances to develop the fruits they had set. This was why the beans of the experimental plants were better filled and more heavily stored with fat, while the portion taken up by the coat was less. Therefore, a higher yield of fat was obtained from these plants.

Table 1

Variety	Sets		Number of days from germination to flowering	Total number of flowers per plant	Number of fruits per plant		Yield of air dry matter in g		% of fat in abs. dry seed	Yield of fat per plant	
	Day-length	Gauze screening			developed	underdeveloped and genophores	fruit	vegetative organs		in g	% to control
Virginia Bunch	10 h.	1 layer	34	42	11.0	9.4	14.7	13.4	50.4	4.63	110
		2 layers	35	33	9.2	7.4	11.8	11.7	49.9	3.62	86
	natural	1 layer	37	43	10.4	14.8	13.8	22.4	48.7	3.41	81
		2 layers	38	43	8.4	19.6	9.9	24.1	46.8	2.32	55
	Control										
	natural	no gauze	36	104	13.7	24.1	17.2	24.9	47.8	4.20	100
	10 h.	1 layer	30	80	28.3	11.3	15.3	10.8	52.8	5.95	102
		2 layers	29	74	23.0	9.7	12.9	11.1	53.5	5.09	87
Spanish 0344	natural	1 layer	30	104	26.4	21.6	14.8	16.4	52.5	6.00	103
		2 layers	30	92	23.0	18.8	13.0	17.1	51.6	4.91	84
	Control										
	natural	no gauze	29	204	34.4	40.5	16.4	20.1	51.4	5.81	100

Screening with one or even two layers of gauze had no influence on the formation of green mass, the latter being affected only by the photoperiod: the short-day plants were much lower in green mass than were the controls.

The fact that in the shade sufficient amount of organic matter, both vegetative and reproductive, is stored up by *Arachis*, permits to class it with ombroliferous plants. It has been pointed out by Lubimenko that the light minimum for photosynthesis to take place is far lower in shade-loving than in light-loving plants. As a reason for this he suggests a stronger concentration of chlorophyll in the plastids of the former, which increases their absorptive power.

This assumption was confirmed by the estimation of chlorophyll in the leaves of peanuts grown under natural and reduced illumination, which was made by the end of the growing period. The shaded plants were higher in chlorophyll in all cases. The results of these tests are given in Table 2.

Table 2

Variety	Sets		Chlorophyll content (in g) per 1 kg of leaves	
	Daylength	Gauze screening	raw weight	abs. dry weight
Virginia Bunch	10 h.	1 layer	2.85	9.69
		2 layers	3.75	11.78
	natural	1 layer	3.33	10.88
		2 layers	3.53	11.80
		Control		
Spanish 0344	natural	no gauze	2.14	7.46
	10 h.	1 layer	2.14	7.23
		2 layers	2.22	8.44
	natural	1 layer	2.40	7.16
		2 layers	2.45	9.48
		Control		
	natural	no gauze	1.46	4.54

It should be noted that the content of chlorophyll was always higher in the late variety Virginia Bunch than in the early variety Spanish 0344. This may be explained in terms of their hereditary qualities.

In view of their response to shading, under normal daylength, peanuts may be referred to ombroliferous plants; it may therefore be recommended to revise the conditions of their cultivation. The question arises: whether a denser planting of peanuts will not possibly bring higher yields per acre.

The fact that the plant does well in the shade can be turned to good account in the case of mixed plantings. This has also been pointed out in literature. Thus, Chevalier [1] reports the possibility of planting peanuts with some other crops (maize, millet and potatoes). In my opinion this is possible only if the attendent culture bears a moderate leaf area and if sufficient amounts of nutrient matter and water are available in the soil. The latter condition is also essential in dense planting experiments.

Leshaft Institute for Natural Sciences.
Laboratory of Plant Physiology. Leningrad.

Received
10. I. 1941.

REFERENCES

[1] A. Chevalier, Revue de Bot. Appl. et de Culture Tropicale, 14, 156.

Translated by D. Rakhmanov.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

PLANT PHYSIOLOGY

CARBOHYDRATE METABOLISM IN ALPINE GRASSES IN CONNEXION WITH THE ADAPTATION PROBLEM

By S. O. GREBINSKY

(Communicated by A. A. Richter, Member of the Academy, 6. II. 1941)

Proceeding farther in our studies on the biochemistry of alpine plants (1), we made a closer inquiry into their carbohydrate metabolism. The analyses for carbohydrates were made according to the method described (1) in plants collected in the humid mountains of the Ala-tau, near Alma-Ata in 1939 and 1940.

As is shown by the figures referred to below, the plants that were found to have increased strongly in sugars all belonged to cultivated varieties which had been brought to the mountains for the first time. In the local wild forms the storage of sugars was much less pronounced (Table 1).

Table 1

Species	Developmental phase	Elevation in m	Sugars		Rest	Total	Insoluble carbohydrates	Total carbohydrates
			reducing	sucrose				
Wheat *erythrospermum* 0341 (cultivated)	Tillering	800	4.48	5.04	0.61	10.13	12.11	22.24
		1520	2.83	7.34	2.33	12.50	8.50	21.00
		1980	2.91	7.69	2.20	12.80	7.65	20.45
		3100	4.34	14.41	1.38	20.13	1.64	21.77
	Heading	1980	3.52	5.23	2.77	11.52	22.12	33.64
		2400	7.41	10.21	0.48	18.10	15.16	33.26
Winter rye (cultivated)	Tillering	1980	2.13	6.23	0.60	8.96	11.18	20.14
		2400	3.78	11.94	0.60	16.32	4.38	20.70
		3100	2.80	12.01	0.47	15.28	1.61	16.89
	Heading	1980	3.93	6.85	0.84	11.62	11.73	23.35
		2400	2.97	14.28	1.10	18.35	5.46	23.81
Dactylis glomerata	»	1000	3.95	4.92		8.87	14.75	23.62
		1520	3.04	5.08		8.12	13.20	21.32
		2400	2.75	6.88		9.63	10.78	20.41

Reduced accumulation of soluble sugars in wild cereals is their hereditary quality which they retain even when moved into a less congenial environment. This conclusion is supported by the carbohydrate analyses made on many plants from the collection of B. A. Keller, after they had been grown as high as 3100 m in a cold plain near the glacier of Tuuk-su in 1939. These plants adapted themselves to the hard conditions of the highland with great difficulty and by the end of the growing period, late in August 1939, had reached a height of several cms only, remaining in the phase of tillering. The winter forms of that collection outlived winter, and by the end of June

1940 attained a considerable height (15—25 cm on the average, as against 4—6 cm in 1939); some of them even proceeded to heading. No formation of seeds took place, however.

It has been revealed by analyses that the cereals tested had stored comparatively little sugar, but contained high amounts of reserve insoluble carbohydrates. Their store of starch was not great (0.8—1.8%), and from this it may be concluded that the conditions, under which they had been growing, favoured the accumulation of the hemicellulose type of carbohydrates. By analyzing a great many wild cereals in the phase of heading collected in this plain, we found them, too, to store great quantities of insoluble polysaccharides of the hemicellulose type.

It will be obvious that wild forage grasses growing under the severe conditions of an alpine climate fail to show the carbohydrate reaction typical of cultivated grasses, which consists in an outstandingly high accumulation of sugars. On the contrary, all the wild grasses show an unmistakable predominance of reserve insoluble carbohydrates over the sugars, as is brought out by the data of Table 2.

Table 2

Storage of Carbohydrates by Cultivated and Wild Cereals in Tuuk-su at a Height of 3100 m (Wt % in dry matter)

Plant	Developmental phase	Date of collection of samples	Soluble carbohydrates	Insoluble carbohydrates	Total	Ratio of soluble to total in %
Plantations of the Academy of Sciences of the USSR						
Poa bulbosa	Tillering	Aug. 23 1939	7.25	21.70	28.95	25.04
» »	Heading	July 23 1940	5.86	12.84	18.70	31.34
Bromus squarrosus	Tillering	Aug. 23 1939	9.91	17.81	27.72	35.75
» »	Heading	July 23 1940	4.13	13.80	17.93	23.04
Aegilops crassa	Tillering	Aug. 23 1939	8.78	15.11	23.89	36.70
Agropyrum repens	»	Aug. 23 1939	6.33	18.90	25.23	25.10
» »	»	July 23 1940	5.23	11.27	16.50	25.23
Dactylis glomerata	»	Aug. 23 1939	7.03	9.95	16.98	41.41
Avena sp.	Stalk formation	Aug. 23 1939	6.78	9.90	16.68	40.65
		July 23 1940	2.67	10.97	14.64	18.23
Wild plants						
Poa pratensis	Heading	July 20 1939	13.54	18.50	32.04	33.57
» »	»	July 23 1940	5.23	16.88	22.11	23.65
Poa relaxa	»	July 20 1939	13.34	15.35	28.69	46.50
» »	»	July 23 1940	6.69	14.31	21.00	31.86
Trisetum sibiricum	»	July 20 1939	8.47	14.93	23.40	36.10
Festuca sulcata	»	July 23 1940	6.67	9.54	16.21	41.15
Festuca spicatum	»	July 23 1940	8.30	13.70	22.00	37.73
Poa alpina	»	July 23 1940	9.16	15.56	24.72	37.05
Poa sp.	»	July 23 1940	3.58	11.16	14.74	24.29
Phleum alpina	»	July 23 1940	6.24	16.76	23.00	27.15
Festuca ovina	»	July 23 1940	8.30	11.17	19.47	42.63

At the same time in the cultivated grasses grown under conditions of Tuuk-su sugars (13.05—20.13%) were predominant over insoluble carbohydrates (1.61—7.87%).

Further investigations have revealed that winter varieties of wheat and rye show a characteristic shift in their carbohydrate metabolism, after they have passed winter at a height of 3100 m in the above locality. On the second year of life such plants considerably increase in reserve insoluble carbo-

hydrates and decrease in sugars. Hence the adaptation of these plants to alpine conditions, their return to the feral state, is attended by such changes in carbohydrate metabolism, as bring them closer in composition to the wild grasses.

Table 3

Daily Variations in the Accumulation of Carbohydrates by Cultivated and Wild Cereals in Tuuk-Su (Wt % in dry matter)

Plant	Hours of the day	Soluble carbohydrates	Insoluble carbohydrates	Total	Ratio of soluble to total in %
Winter wheat	8	7.39	8.01	15.40	47.99
	13	10.15	7.65	17.90	56.69
	18	6.91	9.96	16.87	40.96
Winter rye	8	7.15	10.20	17.35	41.22
	13	10.80	8.93	19.73	54.72
	18	9.17	9.68	18.85	48.64
Bromus inermis	8	5.86	13.54	19.40	30.21
	13	8.25	13.18	21.43	38.50
	18	6.69	12.91	19.60	34.12

An increased accumulation of soluble carbohydrates in the midday hours seems to be a characteristic feature of alpine plants and was reported also by Marthaler (2) for the Bavarian Alps.

It will be apparent that the exceedingly high storage of sugars in response to low temperature set up in the process of hardening is by no means a universal reaction for all cereals. Unlike the cultivated cereals, the wild ones are characterized under conditions of an alpine climate by the prevalence of reserve insoluble carbohydrates of the hemicellulose type. On that ground the wild cereals can be referred, as regards their reaction to chilling, to the group of plants established by Rosa (3) (cabbage etc.), which on hardening store swelling carbohydrates of the hemicellulose type.

The adaptation of cultivated cereals (winter varieties of rye and wheat) to the cold climate of high regions tends even to reduce the storage of sugar, instead of increasing it, whereas the amount of the hemicellulose type of carbohydrates is increased.

While holding to the modern conceptions as to the rôle of protective substances in cold-resistance, we arrive at a conclusion that in the cereals growing in high regions, there are two types of adaptive carbohydrate reaction to chilling: a) storage of sugars and b) storage of swelling carbohydrates of the hemicellulose type. In the course of adaptation of cultivated cereals the former weakens, whereas the latter gains in strength.

Among other wild grasses growing in Tuuk-su, we found some characterized by high contents of sugars (e. g. *Ranunculus Alberti*—22.03%, *Euphorbia alatavica*—15.82%), while others contained much hemicellulose (*Cobresia capillifolia*—14.32%, *Erigeron auranticum*—9.03% etc.).

Thanks are due to undergraduates Kolossova and Loghinova for their assistance in this investigation.

Department of Plant Physiology.
Kirov University in Kazakhstan.

Received
27. I. 1941.

REFERENCES

[1] S. O. Grebinsky, C. R. Acad. Sci. URSS, XXIV, 517 (1939). [2] H. Marthaler, Jahrb. Wiss. Bot., 87, 267 (1938). [3] J. T. Rosa, Miss. Agr. Expt. Sta. Res., Bull. 48 (1921).

Translated by D. Rakhmanov.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

ICHTHYOLOGY

SOME REGULARITIES OF GROWTH AND FORMATION OF EXTERNAL CHARACTERS IN THE CULTIVATED CARP

By K. GOLOVINSKAYA

(Communicated by I. I. Schmalhausen, Member of the Academy, 21. I. 1941)

The present paper contains the principal results of observations on growth and formation of external characters (exterior), of the scattered carp during the first two years of its life ([5–7]). As indices of growth were used the growth constant K and specific growth rate C_v ([1, 11, 14]). The investigations were carried out during the periods of intensive growth, viz., in the first year from the moment of transfer from breeding ponds till wintering, and in the second year, from the end of wintering till fall fishing. The values of K and C_v calculated on the basis of weight for both periods are as follows:

First summer—K from 2.45 to 3.89; C_v from 0.032 to 0.049.

Second summer—K from 4.73 to 12.68; C_v from 0.012 to 0.031.

As was to be expected, the specific growth rate ([14]) showed a decrease in the second year, while the growth constant increased. These differences are even more pronounced in observations on growth within both periods.

The indices K and C_v undergo some changes during the summer. Usually, growth is most intensive early in the season, when K and C_v may reach values of 7.6 and 0.2, respectively, in the first summer and of 36.2 and 0.1, respectively, in the second. Later on the growth slows down, coming almost to a standstill in the fall. The changes of K and C_v are related to the conditions of life, varying during the summer. This was particularly well demonstrated in experiments with heavy stocking and artificial feeding (Table 1).

In the beginning of the experiment and up to July 11, when natural food was plentiful, values of K and C_v were very similar in all ponds notwithstanding differences in respect to density of stocking. By July 21 K and C_v decreased abruptly in heavily stocked ponds. On July 23 feeding was started in ponds 25 and 28; the result was that K and C_v increased at first in pond 28, and somewhat later in pond 25. Till September the values of K and C_v remained at a higher level as compared with the two other ponds, where both indices were very low. Later on the difference was levelled, since with the cooling of water the fish ceased to feed (Table 2).

On May and early June at low water temperatures the carp took artificial food but poorly, and the values of K and C_v were higher in the control pond 8. Till middle July both series differed little from each other in respect to K and C_v. But beginning with the second half of July, when the carp began to feed actively, a considerable increase of K and C_v was observed in ponds with artificial feeding; in controls the rate of growth decreased. During August the growth rate of the experimental series remained on a higher level than in controls. Thus, it is obvious that changes of K and C_v during the summer are related to conditions of life, and, especially, to feeding.

Table 1

Seasonal Changes of Growth Rate and Exterior in the First Summer (Zagorsk, 1939)

Date of weighing	Pond No. 21 Normal stocking, no feeding				Pond No. 24 Threefold stocking, no feeding				Pond No. 25 Threefold stocking with feeding *				Pond No. 28 Tenfold stocking with feeding			
	Mean weight	K	C_v	l/H	Mean weight	K	C_v	l/H	Mean weight	K	C_v	l/H	Mean weight	K	C_v	l/H
June 26	0.054				0.054				0.054				0.054			
July 11	1.28	5.23	0.205		1.29	5.24	0.206		1.57	5.56	0.218		1.36	5.33	0.209	
July 21	9.63	7.62	0.201	2.53	3.59	3.87	0.102	2.63	3.86	3.40	0.090	2.64	2.09	1.61	0.042	2.76
July 31	25.20	4.60	0.094		6.54	2.87	0.060		6.25	2.31	0.048		3.31	2.20	0.046	
Aug. 11	30.79	1.06	0.018		9.77	2.32	0.040		11.57	3.57	0.062		6.70	4.09	0.071	
Aug. 21	32.84	0.46	0.007		8.90	−0.58	−0.009		15.80	1.94	0.028		8.04	1.13	0.017	
Sept. 1	34.26	0.26	0.003	2.71	8.73	−0.12	−0.002	2.79	18.68	1.04	0.013	2.61	9.20	1.31	0.017	2.77
Sept. 11	36.43	0.56	0.006		9.35	0.62	0.007		18.87	0.10	0.001		10.26	0.74	0.008	
Sept. 20	37.43	0.30	0.003		9.01	−0.41	−0.004		21.32	1.34	0.013		10.20	−0.07	−0.001	
Oct. 15/20	36.88	−0.07	−0.001	2.76	9.24	0.11	0.001	2.78	21.73	0.08	0.001	2.69	10.67	0.16	0.001	2.80
During the summer		3.32	0.045			2.61	0.035			3.00	0.040			2.62	0.035	

*Feeding was started on July 23. Food composition: 65% of silk-worm pupae and 35% of grain waste and rape-cake.

Table 2

Seasonal Changes of Growth Rate and Exterior in the Second Summer (Zagorsk, 1939)

Date of weighing	Pond No. 8 Normal stocking, no feeding						Ponds Nos. 10, 12, 13, 15 Threefold stocking with feeding *					
	Weight			Exterior			Weight			Exterior		
	Mean weight	K	C_v	l/H	K by length	K by height	Mean weight	K	C_v	l/H	K by length	K by height
May 15—16	18.12			2.76			19.28			2.76		
June 3—5	33.35	10.20	0.029	2.65	2.40	3.13	29.60	7.65	0.022	2.62	2.50	2.99
June 17	70.63	22.89	0.062	2.47	7.49	9.45	48.98	10.11	0.028	2.66	3.04	3.04
July 3—4	149.17	16.67	0.044	2.58	6.05	5.07	109.50	16.81	0.044	2.68	5.39	5.14
July 16	207.50	10.93	0.028	2.57	4.35	4.56	140.55	10.08	0.021	2.61	3.35	4.26
Aug. 3	310.88	9.38	0.023	2.55	2.50	2.64	258.78	13.78	0.034	2.56	3.61	3.80
Aug. 16—17	351.36	3.73	0.009	2.65	1.98	0.78	302.70	5.70	0.013	2.62	2.55	2.13
Sept. 3	394.00	2.81	0.006	2.63	1.15	0.86	359.30	3.96	0.009	2.58	1.29	2.02
Sept. 16	412.17	1.75	0.004	2.77	1.39	0.04	366.03	0.66	0.002	2.68	0.85	−0.60
During the summer		10.04	0.025		3.34	3.33		9.49	0.023		3.20	3.24

* Food composition: rape-cake with addition of grain waste.

The cause of differences of the growth constants in the first and second summers remains as yet obscure. The question arises whether these differences are determined by peculiarities of growth inherent in the ages investigated, or by environmental conditions.

It is a known fact that in tropics, where growth proceeds continuously, the carp, in its first year, reaches a weight of 1500 g or more ([2], [4]). In the Ukrainian and other southern districts of the USSR, the year's fry can be reared to a weight of 500 g or more ([12], [8]). In such cases values of K and C_v acquire an entirely different numerical expression. So e. g. if carps in their first summer attain a weight of 500 g, the values of K and C_v will be of the order of 5 and 0.07, respectively; if, in the second summer, these carps reach a weight of 2000 g (which is practically possible), K and C_v will have values of the order of 5 and 0.01. Differences between the growth rate of both ages will be greater than in our experiment, whereas the growth constants will be identical. These considerations lead to assume that differences between growth constants of the first and second summer are determined by the specific conditions of our northern climate. A delay in growth in the first year may result in a violent outbreak of growth in the following spring. The fact of a high intensity of growth in backward carps is corroborated by a series of literary data ([2],[9]) and our own observations. That carps which were in arrears at the beginning of the summer do not always overtake fishes with a greater weight, especially when reared together ([13]), may be explained by the better opportunities of large fishes in the struggle for food, during our short vegetative period. Thus, we come to the conclusion that differences in growth constants are determined not by peculiarities of the age, but by environmental conditions prevailing at the time of growth.

Seasonal changes of the rate of growth have a great influence on the formation of the external characters of the carp. Let us compare the indices K and C_v by weight with the index l/H (l—body length, measured from the tip

of snout to the end of the scaly covering, H—greatest height of the body). In the most high-bodied cultivated «races» of carp this index falls to 2.0, whereas in wild carp it is as high as 4.0. In the summer l/H varies according to the changes of the rate of growth: it decreases (the exterior «improves») during periods of intensive growth and increases (the exterior «changes for the worst») when the rate of growth begins to decline. This relation is particularly obvious in experiments with heavy stocking and artificial feeding (Tables 1 and 2). In general the periods of high growth rate correspond to marked changes in the relationship of different body parts and their relative growth rates ([10]). Our investigations on the growth of carps in length and height show that in periods of intensive growth the values of K and C_v for growth in height are higher than the corresponding values for linear growth, and vice versa. This is fully confirmed by our observations of 1939 (Table 2). In experimental carps and in controls the relationship of growth constants for length and height was different, and the nature of the difference was in concordance with the differences in the general rate of growth. The predominance of growth in height is evidently related to periods of intensive feeding. At other times the food consumed is principally used for the formation of the skeleton, and the growth in height, mainly related to accumulation of muscular tissue, begins to lag. Therefore a change of the exterior «for worse» may be taken as a signal of a decreasing growth rate, often at a time when no other alarming symptoms are noticeable.

On the other hand, our data indicate that great weight is not always related to better exterior. Such examples are found in the investigations of Bushkiel ([2–4]). This is probably due to the changes of the rate of growth which occur during processes of development. Weight and exterior attained by a certain time often depend upon the distribution of these changes in time. This was clearly demonstrated in experiments with feeding, when the increase in growth rate after a delay in early summer was strong enough to improve the exterior, but could not compensate for the backwardness in respect to weight. Such an explanation is, however, far from exhaustive. The relation between the rate of weight increment and the formation of the exterior is a very complicated one, it depends on a whole complex of conditions, including the influence of environment and the expression of hereditary properties of the organism.

Thus, we see that environmental conditions exert a very strong influence on the regularities of growth and formation of the exterior of the carp. This fact is of great importance for the development of methods of breeding in carp cultivation and should be taken into account in the working out of these methods.

Department of Genetics and Breeding.
Institute for Pond Fishery. Moscow.

Received
21. I. 1941.

REFERENCES

[1] S. Brody, Univ. of Missouri Agric. Exp. Stat. (1927). [2] A. Buschkiel, Int. Rev. d. ges. Hydrob. u. Hydrogr., **27** (1932). [3] A. Buschkiel, ZS. f. Fischerei, **36**, H. 1 (1938). [4] A. Buschkiel, ibid., **31**, H. 4 (1933). [5] K. Golovinskaya, C. R. Acad. Sci. URSS, XXVIII, No. 6 (1940). [6] V. Kirpichnikov, ibid. XIV, No. 1 (1937). [7] В. Кирпичников, Биол. журн., VI, № 3 (1937). [8] В. Мовчан, Н. Туркевич и Т. Красовская, Рыбн. хоз. СССР, № 2 (1933). [9] E. Probst und E. Walter, Fisch. Zt., **35**, No. 22 (1932). [10] К. Скуфьин. Бюлл. о-ва естествоисп. при Ворон. гос. ун-те, III, вып. 1 (1939). [11] I. Schmalhausen, Roux' Arch. f. Entw.-Mech., 190 (1927). [12] К. Ушаков, Рыбн. хоз. СССР, № 8—9 (1932). [13] Б. Черфаси К. Будников, там же, № 8—9 (1932). [14] И. Шмальгаузен, Сб. Рост животных (1935).

Translated by E. Alexandrova.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

MICROBIOLOGY

DESTRUCTIVE CHANGES OF THE PROTOPLASM IN THE COURSE OF LYSIS IN THE SPECIES *FUSARIUM*

By W. F. ALTERGOT, K. S. LAVYGHINA and O. P. KUVSHINOVA

(Communicated by A. A. Richter, Member of the Academy, 10. II. 1941)

Microscopical investigations of the structure of the protoplasm of vegetable and animal cells ([2], [3], [12], [13]) served as a basis for the creation of various theories of protoplasm structure (reticular, foam-like, fibrillar, granular). At present there is reason to believe that in many cases these microstructures of the protoplasm should be assumed to reflect the cellular constitution appearing under the influence of various natural factors; in some instances,

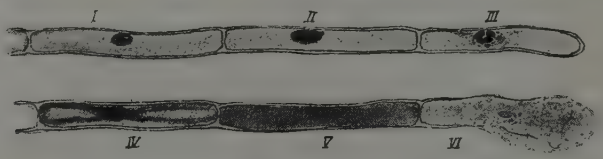


Fig. 1. Scheme of nuclear disintegration during cell lysis of *Fusarium graminearum* Schw.

however, these structures are nothing else than arteacts caused by some rough external influence ([6]).

The aim of the authors of the present investigations was to follow systematically, with the help of the microscope and under the application of microchemical reactions, the state of complex formations and protoplasmic structures in the cells of *Fusarium* mycelium undergoing gradual lysis. These data entitle one to form a true opinion on the extent of destructive processes in the protoplasm, and to understand more thoroughly the sources of formation of the lytic agents already investigated ([1], [4]).

Lytic and non-lytic strains of *Fusarium niveum* Smith., *Fusarium lini* Bolley, *Fusarium graminearum* Schw., *Fusarium solani* (Mart.) App. et Wr. were the object of our investigation. The fungi were cultivated on Waxman's and Richard's solid nutrient media, as well as on must agar medium and others at 27 — 28° C. Spontaneous lytic strains and such where lysis was excited by means of special nutritive and temperature conditions, were studied ([1]). As lysis gradually proceeded, mycelium preparations were made, and the state of the nuclei in the cells [by means of Feulgen's reaction ([11])], volutin granules [after Zikes ([14])] and accumulation of lipoid bodies [after

Meyer ([8], [9])] were examined. The application of Feulgen's method of nuclei staining to the object investigated had required the elaboration of a special technique of fixation and mounting of the material on the slide. Mycelium particles of the colony under investigation were stretched carefully with preparation needles in a drop of distilled water, and gradually passed through rising concentrations of ethyl alcohol into 96° alcohol. The investigations under the microscope were carried out from the moment of macroscopically detectable colony growth.

Table 1

Decomposition of *Fusarium graminearum* Schw. Nucleus in the Course of Lysis

Age of colony in days since the moment of seeding	External view of the colony	Microscopic picture of mycelium in a squeezed drop of water	Staining of nucleus after Feulgen
		Lytic Strains	
3	Fluffy aerial mycelium	Normal cells of the mycelium possessing homogeneous protoplasm; scarce granules	Cell nuclei round; stained violet (Fig. 1, I)
8	Mycelium adhering to the substratum either around the edges or in the centre of the colony. Formation of mucus	Cells atypical in shape; stressed granulation; protoplasm strongly vacuolised	Along with normal ones deformed nuclei splitting into fragments, or nuclei with blurred contours are present (Fig. 1, II, III)
17	Scarce islets of fluffy mycelium; almost the entire colony became mucous	Normal cells are few. The hyphae form irregular accumulations. Many hyphae fragments with blurred contour; a large quantity of extracellular granules	Normal nuclei almost absent. For the most part deformed nuclei at various stages of destruction; nuclear fragments, nuclear matter spread diffusely throughout the cell, the cell contents stains evenly violet (Fig. 1, III, IV, V)
33	All colonies mucous; many of them underwent complete lysis	Rests of hyphae, shapeless accumulations, abundant granulation; many chlamydo- and macrospores	Nuclei absent from yet intact cells; diffuse colouring of the contents. Nuclei formed in chlamydo- and macrospores begin to split and lose the preciseness of their contours. Part of the extracellular granulation is stained slightly violet (Fig. 1, V, VI)
		Non-lytic Strains	
33	Normal fluffy mycelium	Cells of common shape, containing minute granules	Nuclei of distinct shape, staining reddish-violet (Fig. 1, II)

Table 1 contains some data concerning the lysis of the nucleus, while Table 2 shows changes in the volutin granules and accumulation of lipoid

bodies in *Fusarium niveum* Smith. and *Fusarium graminearum* Schw. The results of the investigation of other species are identical with the previous ones, though some of the phases studied by us were delayed. Our observations revealed a gradual and profound disintegration of the cell nuclei of *Fusarium* during the colony lysis; at first the outlines of the nucleus are blurred, ending in a complete dissolution of the nuclear material in the protoplasm. Caryolytic phenomena are well known for various pathological states of the cell ([7]), as well.

Table 2

Disappearance of Volutin Granules and Accumulation of Lipoid-like Substances in the Protoplasm of the Cells of *Fusarium niveum* Smith.

Age of colony in days since the moment of seeding	Volutin granules (staining after Zikes)	Lipoid-like substances (staining after Meyer)
	Lytic Strains	
4	The cell protoplasm stains azure. Numerous minute volutin granules, stained dark blue	Cell protoplasm stains light azure. Minute lipoid inclusions are few; they stain bright-red
8	Protoplasm stains azure. Singular minute volutin granules stained violet or violet-red are present in rare hyphae cells	Protoplasm stains azure. Number and size of lipoid inclusions increased; they stain red
12	Protoplasm of few cells stains azure, most cells colourless. No volutin granules present	But few cells retain light-azure staining of their protoplasm. Most are filled with red lipoid inclusions of various size and shape
	Non-lytic Strains	
12	Cell protoplasm stained light-azure. Numerous volutin granules stained reddish-violet	Cell protoplasm stained light-azure. Minute lipoid inclusions are few in number, stain red

Metachromatin protoplasm formations—volutin granules—are rapidly disappearing at the first stages. Accumulating of the lipoid-like inclusions is connected with the lipophanerose rising described first for the muscular tissue ([10]). Phenomena of caryolysis, lipophanerose, disappearance of volutin granules found during the lysis, have been reported in the case of unfavourable external influences on the cell ([7], [9], [14]).

The results of the present investigation show that the lysis of the organism of a fungus is a process affecting and materially altering the nucleo- and lipoproteid complexes of the cell protoplasm.

And if the recent facts and presumptions concerning the intimate connexion existing between such cell regulators as vitamins, ferments and hormones, and the basic constituents of the protoplasm hold true ([5], [6]), it should be assumed that the lytic factor ([1], [4]), accumulating in the course of lysis and approaching by its biological action the group of bios substances ([1]), is also connected with compound protoplasmic complexes, participates and is liberated in their decomposition.

Department of Microbiology.
Saratov State University.

Received
10. II. 1941.

REFERENCES

[1] W. Altergot, C. R. Acad. Sci. URSS, XXIV, No. 4 (1939). [2] R. Altmann, Die Elementarorganismen und ihre Beziehung zu den Zellen (1894). [3] O. Bütschli, Untersuchungen über mikrosk. Schäume und das Protoplasma (1892). [4] A. Werner and W. Altergot, C. R. Acad. Sci. URSS, XV, No. 4 (1937). [5] А. Р. Вернер, Вещества клеточного деления и их физиолог. значение, рукопись (1940). [6] А. Р. Кизель, Химия протоплазмы (1940). [7] W. W. Lepeschkin, Zell-Nekrobiose und Protoplasma-Tod, Protopl. Monogr., XII (1937). [8] Meyer, Die Zelle der Bakterien (1912). [9] A. Meyer, Morphol. und physiol. Analyse der Zelle der Pflanzen und Tiere (1920). [10] A. Noll, Arch. Anat. u. Phys., 35 (1913). [11] R. Feulgen, Handb. biol. Arbeitsmeth., Abt. 5, T. 2, H. 4, 6—9. [12] N. Flemming, Zellsubstanz, Kern und Zellteilung (1882). [13] Frommann, Jenaische ZS. f. Med. u. Naturw., 9 (1875). [14] H. Zikes, Zntrbl. f. Bakt., Abt. II, 57, 1/3 (1922).

Translated by M. Peshkov.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

MICROBIOLOGY

DEVELOPMENT OF AZOTOBACTER IN THE SOILS OF ARID STEPPES AND DESERTS

By N. N. SUSHKINA

(Communicated by B. A. Keller, Member of the Academy, 23. XII. 1940)

The development of azotobacter in soils is controlled by a variety of conditions. Chemical, biological and climatic factors combine to influence it. Among them are acidity of the soil, its humidity, the amount and composition of organic matter it contains, the attendant microflora, etc. Which of the factors are the most important, depends upon the climatic zone.

I have analyzed about 1000 samples from the dry steppe and desert zone, including chestnut soils, brown soils, grey soils, meadow soils, alkali soils from steppe and meadow land (the solonets), saline soils (the solonchak), virgin meadow soils liable to occasional flooding, sandy soils and sands. The analyses have shown that in all these soils the development of azotobacter is decisively affected by the content of moisture and the nature of the vegetation cover.

Unlike most of the northern soils which are acid, those of the dry steppe and desert zone are either neutral or alkaline. Here therefore there is no influence of acidity which is a strong barrier to the dispersal of azotobacter in the soils of the podzol zone. The process of soil formation goes on either under such kind of vegetation as usually covers arid areas in steppes, semideserts and deserts, or, when there is lowering of ground and more surface water is available or the water-table is nearer, the vegetation cover is of the meadow or solonchak-meadow type.

In the first case the absence of azotobacter is a general rule and is due to the lack of moisture in the soil. But in the meadow soils, where the conditions of moisture supply are more favourable, the azotobacter is often encountered.

With regard to the vegetation covering dry steps and deserts, it may as well be stated that only a depressing influence on the development of azotobacter is exercised by the rhizosphere of their plant communities, whereas in soils of the meadow type the rhizosphere is a favourable medium for the development of azotobacter.

We find no azotobacter in the virgin soil—chestnut coloured, brown, or grey—within arid areas of the zone in question. Nor does it occur in alkali steppe soils bearing their typical vegetation. But it does occur in meadow soils, alkaline, saline and virgin, and also in sands when they are well provided with water and covered with a meadow type vegetation composed of more or less hygrophytous species. After irrigation, however, the azotobacter

will never fail to appear in those arid areas of the dry steppe and desert zone, too.

Across the profile of alkaline meadow soils the distribution of azotobacter in this zone is confined to horizons of higher water holding power B_1 and B_2. In the dry horizon A_1 azotobacter appears only in response to watering. This is illustrated by several examples in the adjoined table.

Soil profile	Place of sampling	Kind of soil	Horizon and depth in cm	Growth of azotobacter*	Vegetation cover
66	Kulundin steppe. Lake Burlinskoye	Alkali meadow soil, slightly saline	A_1 0—5 B_1 11—20 B_2 20—30 50—60 150—160	0 ++++ +++++ ++ 0	Against the background of *Atropis* sp. there are many *Artemisia maritima salina;* some *Statice* sp. and many lichens
35	South-eastern part of Caspian plane between the rivers Ural and Emba	Alkali meadow soil, moderately saline	A_1 2—5 B_1 6—12 B_2 20—35 40—45 70—80 120—130	0 +++ + 0 0 0	Saline soil vegetation
30	Kirghizia, 40 km of Frunze	Alkali meadow soil, strongly saline	A_1 0—3 B_1 5—10 B_2 25—30 65—70 150—155	0 ++ +++ + 0	Against the background of *Artemisia maritima salina* there occur: *Atropis distans, Glycyrrhiza glabra, Camphorosma* sp., *Poa bulbosa*

* Scale of growth rate by azotobacter: +++++ strong; ++++ higher than medium; +++ medium; ++ lower than medium; + weak; 0 no growth.

As for the high resistance of azotobacter to drying, which some of the authors have reported in pure cultures, it is hard to believe that it should be as strong in the soil. And what is more, the quantitative side of the question must not be left unaccounted, as is all too often the case. In testing samples of alkali soils from the Lower Volga I found that azotobacter made very intensive growth in those soils from the very start and was losing its power of growth only very slowly, while being kept in air dry condition at the laboratory. Eventually all growth ceased, but this was after 5 years of standing. And throughout these 5 years or nearly so the gradual dying away of the microbe had certainly remained undetectable by qualitative analysis, which whenever made would have only recorded the presence of azotobacter in the soil.

Interesting data in support of this statement are to be found in a paper by Jones Dan. When keeping azotobacter cultures dry and without reinoculation, he found after seeding them on nutrient media that:

The culture kept during	gave
16 days	9 000 colonies
5 months	5 000 »
1 year and 4 months	2 200 »
2 years and 2 months	60 »

So the cells of azotobacter had been gradually dying away and the death rate had sharply increased in time. It will be obvious that most of the microbe's cells if not all are eventually killed if subjected to drying.

Now, under conditions of a hot and dry climate whose influence on the soil is as long as not to be measured in years, but rather in geological periods, it is only natural to expect a complete destruction of azotobacter, which is not able to form spores and so protect itself from the uncongenial environment.

And if viable cultures of azotobacter obtained after reinoculations from strains kept dry for several years have been reported in literature, this can by no means affect our statement as to the importance of moisture for the dispersal of azotobacter in the soils of the dry steppe and desert zone. Indeed, the development of azotobacter in these soils is so strongly dependent upon the conditions of water supply, which determine here also the character of the vegetation cover, that the presence or the absence of azotobacter can be predicted when the composition of the local flora is known.

Institute of Microbiology.
Academy of Sciences of the USSR.

Received
24. XII. 1940.

REFERENCES

[1] H. Jones Dan, Zbl. Bakteriol. (Abt. 2), 42, 68 (1914).

Translated by D. Rakhmanov.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1941. Volume XXXI, № 3

PALEONTOLOGY

HETEROPOLAR DIVISION OF THE SUTURE LINE IN SOME OF THE LOWER TRIASSIC AMMONITES

By J. N. POPOV

(Communicated by A. A. Borissiak, Member of the Academy, 10. II. 1941)

According to the mode of division of lobes and saddles in ammonites, Wedekind (1) distinguished two types of division of the suture line: unipolar and bipolar. The division of the suture line Wedekind conceived as a process, and therefore the types of division (unipolar and bipolar) were treated

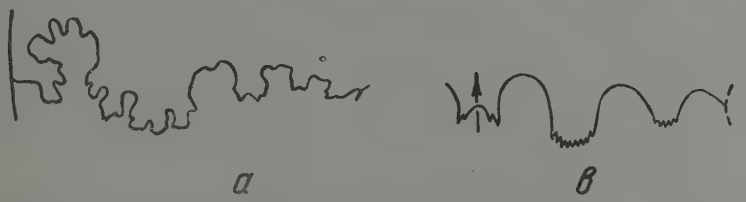


Fig. 1. *a*—Suture line in *Glyptophiceras nielseni* Spath., as an example of unipolar division (sagittal-unipolar); *b*—suture line in *Hoplitoides ingens* Koenen, as an instance of bipolar division (sagittal-bipolar).

by him not in the sense of formal logic, but in the natural history aspect. Consequently, Wedekind makes sharp distinction between the sutural elements and indentations and notches (Inzisionen) in them. He stresses the point that lobes and saddles are «the primary curves of the suture line», whereas Inzisionen—«notches in the lobes and saddles, the notching appearing first in those sutural elements which lie nearest to the plane of symmetry of the shell» (1, S. 192), i. e. division begins always from the external (E) and internal (I) lobe.

And whereas the formation of lobes and saddles is subject to special laws (Lobengesetz), the principles underlying the formation of notches and indentations are entirely different, i. e. those of unipolar and bipolar division. In both cases those elements are the first to undergo division which lie nearest to the plane of symmetry of the shell, other elements following in consecutive order umbilically.

Fig. 1, *a* and *b* are an excellent illustration of Wedekind's conception. In this sense both types of division could be termed sagittal-polar types of division, because of the initial poles of division being arranged near the sagittal plane, or the plane of symmetry of the shell.

However, the unipolar and bipolar types of division of the suture do not cover all modes of division which may eventually develop, as indicated by Diener ([2], [3]). As far as division is concerned, Diener pointed out the suture in *Hedenstroemia muthiana* Krafft ([4], pl. IX, Fig. 7c), not fitting in the system of Wedekind's classification ([2], S. 378), and made the assumption that «the establishment of unipolar and bipolar division does not yet cover the full range of variation of notches in the ammonite sutures» ([3], S. 592). However, the material which was in Diener's possession was inadequate and did not carry sufficient weight.

In 1939 the author described *Anahedenstroemia tscherskii* Popow ([5], p. 77, table 1, fig. 4) from the Lower Triassic deposits of the Okhotsk-Kolyma divide. The suture of this species is shown in Fig. 2.

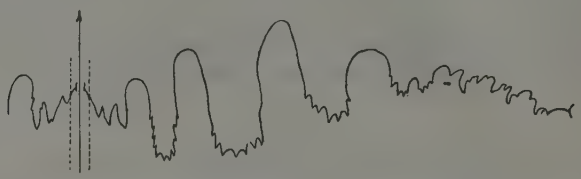


Fig. 2. Suture line in *Anahedenstroemia tscherskii* Popow, as an instance of heteropolar division (sagittal-unipolar and umbilical-bipolar).

The division of the external and two lateral lobes and their corresponding saddles is of the unipolar type, i. e. only the bases of the lobes are serrated, the summits of the saddles are entire. But beginning from the first auxiliary lobe, all the lobes and saddles of the auxiliary row display already bipolar division. The character of division is entirely different from the one in the main specimens. Thus, the suture line of this species from the fam. *Hedenstroemiidae* Waagen combines both the unipolar and bipolar mode of division. Consequently, such division is to be regarded as heteropolar.

The initial poles of the secondary division of the auxiliary elements are situated in the base of the first auxiliary lobe and in the summit of the first auxiliary saddle on each side of the shell, i. e. the division of auxiliary elements in *A. tscherskii* begins not near the sagittal plane, but much more distally. One may say that the auxiliary poles of division lie near the projection of the external margin of the inner volution of the shell on the flank of the outer whorl. For this reason the auxiliary poles of dissection could be called «projection poles».

As a matter of fact, however, the secondary division begins from the umbilical suture. This is proved by the division of the suture in the Olenek specimen of *Hedenstroemia mojsisovicsi* Diener, figured by E. Mojsisovics ([6], pl. III, fig. 13). The suture line in this species, too, undergoes the heteropolar type of division. Besides, both the structure and division of all the elements of the auxiliary line and the inner portion of the suture line show remarkable symmetry, except the inner lobe and the saddles. The axis of symmetry lies on the umbilical suture.

In this case it may be assumed that the secondary division of auxiliary and internal elements of the suture line begins from the umbilical suture on both flanks of the shell, and the poles of division lie in the plane passing through the points of the umbilical constriction on both flanks of the shell. The secondary division of the internal and auxiliary elements in this case may be termed the umbilical-polar type of division.

The above instances of heteropolar division of the suture lines in representatives of the fam. *Hedenstroemiidae* Waagen from the Lower Triassic of north-eastern Asia bear evidence, that the heteropolar division is not a casual occurrence or ill-stated fact, as was believed by Wedekind (S. 536), but is peculiar to many species of the Boreal region of the Lower Triassic.

In the controversy between Diener and Wedekind ([1], [2], [3], [8]) it was Diener who was right. The types of division of the suture in ammonites can be schematized as follows:

A. Sagittal-polar division: a) unipolar division, e. g. *Ophiceras*, *Glyptophiceras*, *Ceratites* and others; b) bipolar division, e. g. *Stephanoceras*, *Hoplitoides*, etc.

B. Umbilical-polar division: a) unipolar division; b) bipolar division.

C. Heteropolar division or combined division (sagittal-umbilical-polar): a) sagittal-polar (unipolar) and umbilical-polar (bipolar) is observed in *Anahedenstroemia tscherskii* Popow and *Hedenstroemia mojsisovicsi* Diener (Olenek specimen).

The B type division in a pure state has not been observed yet and is hardly possible; but there are other types of heteropolar division likely to occur (C).

Besides representatives from the fam. *Hedenstroemidae*, the tendency to heteropolar division is expressed also in *Pseudosagoceras multilobatum* Noetling ([7], pl. XIX—XXVII).

The umbilical seam assumes great importance as the site of formation of new elements of the suture line, as well as new indentations and notches upon them. But if the umbilical suture and the so-called «Umschlag» are the area, where not only new elements, but also indentations in them are forming, the difference between the lobes and notches, established by Wedekind as a matter of principle and strongly disputed by Diener, is obliterated. Every notch with gradual increase in dimensions is transformed into a lobe, and vice versa.

There still remains unclear in what relation to heteropolar division stands the frequently observed asymmetrical structure of the auxiliary elements of the suture line on one and the same suture on different sides of the shell, as observed in *Anahedenostroemia himalayica* Spath., in *Hedenstroemia borealis* sp. nov. and in *Pseudosagoceras multilobatum* Noetl.

Received
10. II. 1941.

REFERENCES

[1] Wedekind, Zentralbl. f. Min., etc., No. 8, 185 (1916). [2] C. Diener, ibid., 374 (1916). [3] C. Diener, ibid., 553, 578 (1916). [4] Krafft and Diener, Palaeontologia Indica, ser. XV, VI, Mem. No. 1 (1909). [5] Ю. Н. Попов, Проблемы Арктики, № 12, 72 (1939). [6] Ed. Mojsisovics, Mém. de l'Acad. de St.-Pétersb., sér. VII, XXXVI, No. 5, 22 (1888). [7] F. Noetling, Palaeontographica, LI, 155 (1905). [8] R. Wedekind, Zentralbl. f. Min., etc., 529 (1916).

Translated by T. Sachs.

Orders to be placed with:
Les commandes sont à adresser à:
Bestellungen sind zu richten an:

MEZHDUNARODNAJA KNIGA

Moscow, Kuznetski Most, 18.

Remittances to be effected to the State Bank of the USSR (UNIO)
Moscow, Neglinnaja, 12.

USA — Four Continent Book Corporation, 255 Fifth Avenue, New York, N. Y.

BELGIQUE — «OBLA», 6/8/10, rue Duquesnoy, Bruxelles.

BULGARIA — T. F. TSCHIPEFF, Boulevard Dondukoff 20/22, Sofia.

CHINA — Fleet's Book Store, 798 Avenue Joffre, Shanghai.

CHILE — Libraria Ibero-Americana de Publicaciones, Moneda 702, Casilla 13201, Santiago.

COLOMBIA — Distribuidora Nacional de Publicaciones Manuel Abondano H. Apartado Nacional 27—29, Bogota.

DANMARK — S. U. V.'s Boghandel, Frederiksborggade, 42, København.

DEUTSCHLAND — KOEHLER & VOLCKMAR, Russland-Abteilung, Leipzig C. 1. Postfach 178.

GREAT BRITAIN — Collet's Bookshop, Ltd., 66 Charing Cross Road, London, W. C. 2.

JUGOSLAVIJA — GECA KON A. G. Postfach 103, Beograd.
— «Slavija» Alex. I. Ivanoff, Krunska, 25, Beograd.

MAGYARORSZAG — Grill'sche Buchhandlung, Dorottya ut. 2, Budapest.

MEXICO — Editorial Popular, Apartado 2352, Mexico D. F.

NEW ZEALAND — Modern Books, 12 Woodward Street, Wellington C 1.

NORGE — JOHAN GRUNDT TANUM, Karl Johans gt. 43, Oslo.
— A/S Narvesens Kioskompani, Postbox 125, Oslo.

PALESTINE — S. ISERSKY, Grusenberg Street 24, Tel-Aviv, P. O. B. 656.
— «MASSADAH», Ltd., Herzl Street, 2, Tel-Aviv.

ROMANIA — «LOUVRE», strada Bibescù Voda 2, Bucureşti.

SUOMI — Akateeminen Kirjakauppa, Helsinki.
— Rautatiekirjakauppa Oy, Köydenpunojankatu 2, Helsinki.

SVERIGE — WENNERGREN-WILLIAMS A. B., Box 657, Stockholm.
— C. E. FRITZE's Kgl. Hofbuchhandlung. Fredsgatan 2, Stockholm.

TÜRKIA — LIBRAIRIE HACHETTE, 469, Avenue de l'Indépendance, Boîte postale Péra 2219, Istanbul.

URUGUAY — Distribudora de Publicaciones, Calle Eduardo Acevedo, 1450, Montevideo.

Prix $ 0.50—

PUBLISHING HOUSE OF THE ACADEMY OF SCIENCES OF THE USSR

JOURNAL OF PHYSICS

Edited by S. I. WAWILOW, Member of the Academy of Sciences.
Editorial Secretary: Prof. S. M. RYTOV, D. Sc.

Journal of Physics deals with problems of theoretical and applied physics.

Journal of Physics is published in English, French and German.

Journal of Physics appears monthly; it is issued in two volumes per annum, 6 numbers per volume.

Subscription price: U. S. $ 8.00 per annum. $ 1.00 single copy.

Orders to be placed with:

SSSR—«Mezhdunarodnaja Kniga», 18 Kuznetski Most, Moskva.
USA—Four Continent Book Corporation, 255 Fifth Avenue New York, N. Y.
BELGIQUE—«Obla», 6/8/10, rue Duquesnoy, Bruxelles.
BULGARIA—«Ruska Kniga», ul. kn. Boris, 2, Sofia. T. F. Tschipeff, Boulevard Dondukoff, 20/22, Sofia.
CHINA—Fleet's Book Store, 798 Avenue Joffre, Shanghai.
DANMARK—S.U.V's Boghandel. Frederiksborggade, 42—1, København.
DEUTSCHLAND—Koehler & Volckmar A.-G. & Co., Russland-Abteilung, Leipzig C. 1. Postfach 173.
GREAT BRITAIN—Collet's Bookshop, Ltd., 66 Charing Cross Road, London, W. C. 2.
JUGOSLAVIJA—Geca Kon A.-G. Postfach 103, Beograd.
MAGYARORSZAG—Grill'sche Buchhandlung, Dorottya ut. 2, Budapest.
NEW ZEALAND—Progressive Books, Ltd., 18 Darby Street, Auckland C. 1.
NEW ZEALAND—Modern Books, 12 Woodward Street, Wellington C. 1.
NORGE—Johan Grundt Tanum, Karl Johans gt., 43. Oslo.
ROMANIA—«Louvre», Strada Bibescu Voda, 2, Bucureşti.
SCHWEIZ—Buchhandlung Stauffacher, Lutherstrasse 6, Zürich.
SOUTH AFRICA—People's Bookshop (Pty.), Ltd. Trades Hall, Kerk Street, Johannesburg.
SUOMI—Akateeminen Kirjakauppa, Helsinki.
—Rautatiekirjakauppa O. Y., Köydenpunojankatu 2, Helsinki.
SVERIGE—Wennergren-Williams A.-B. Box 657, Stockholm.
—C. E. Fritze's Kgl. Hofbuchhandlung, Fredsgatan 2, Stockholm.
TÜRKIA—Librairie Hachette, 469 Avenue de l'Indépendance, Boite postale Péra 2219, Istanbul.
PALESTINE—S. Isersky, Grusenberg Street 24, Tel-Aviv. P. O. B. 658.
—«Massadah», Herzl Street, 2, Tel-Aviv.

EDITORIAL OFFICE: Moscow, 47, 3rd Miusskaya Ulitsa, No. 8, Physical Institute of the Academy of Sciences of the USSR.